68. Arbeitstagung für Allgemeine Algebra

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Complexity of Algebra and Algebra of Complexity

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These are classes of *combinatorial decision problems*, i. e. problems whose input is a finite object (graph, formula, algebra, . . .) and whose question is whether or not a given object possesses a certain property (which usually gives the name to the problem). The answer to each concrete instance of such a problem is either YES or NO.

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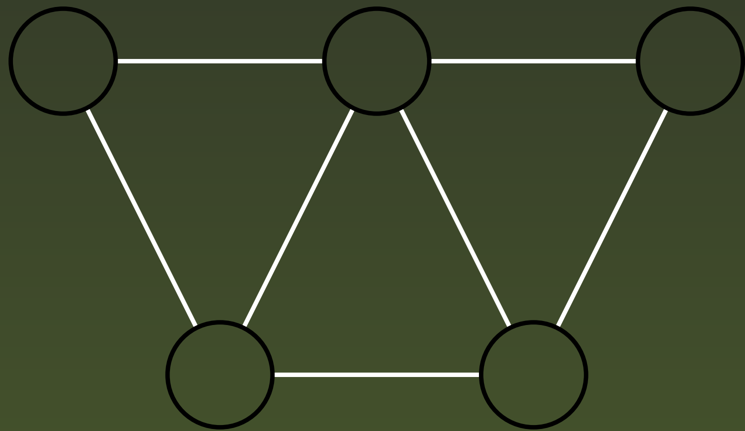

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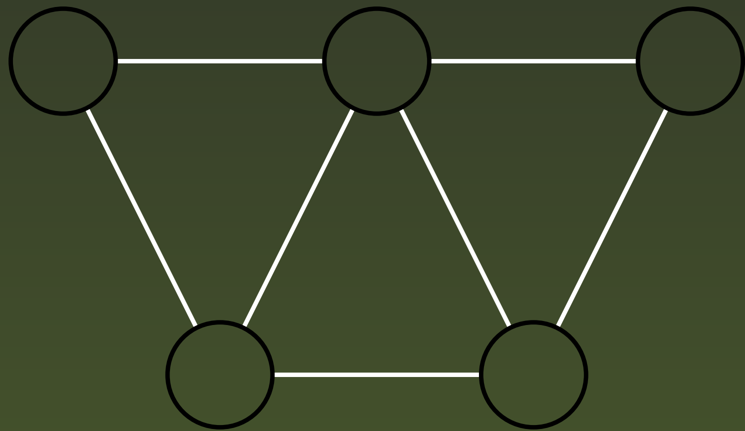

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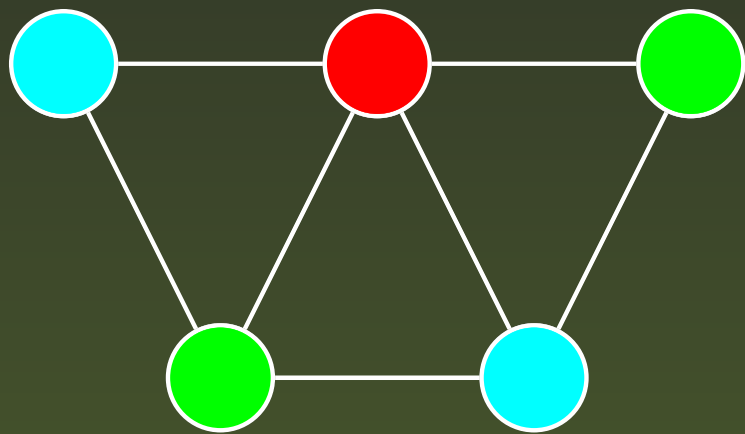

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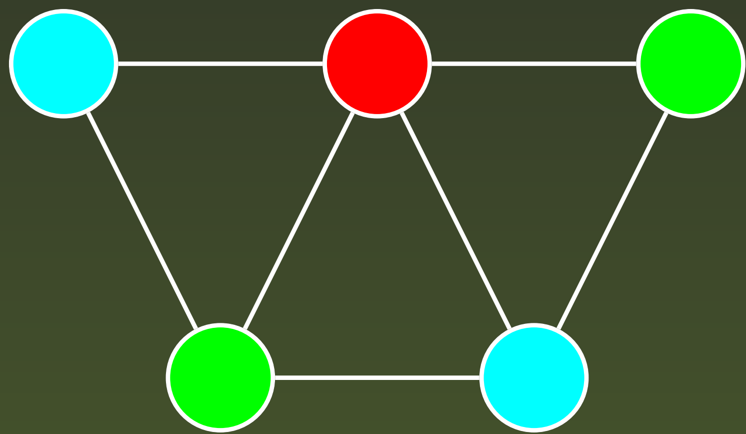

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The Classes P and NP

Arthur, a normal man

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Merlin, a superman

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Example: 3-COLOR is **NP**-complete (Levin, 1973).

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and the latter condition can be algorithmically tested.

But is this an efficient solution? It doesn't seem so — if $|A| = n$ and $|B| = m$, the only bound for the size of $\mathrm{Clo}_{|B|}(\mathbf{A})$ is $n^{(n^m)}$ so the above algorithm requires doubly exponential time (as a function of $|B|$).

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Some important and application-oriented questions of formal language theory lead exactly to the problems of the form “Does a given $\mathbf{B}$ belong to the variety $\mathrm{var}\,\mathbf{A}$?" in which $\mathbf{A}$ and $\mathbf{B}$ are *finite semigroups*.

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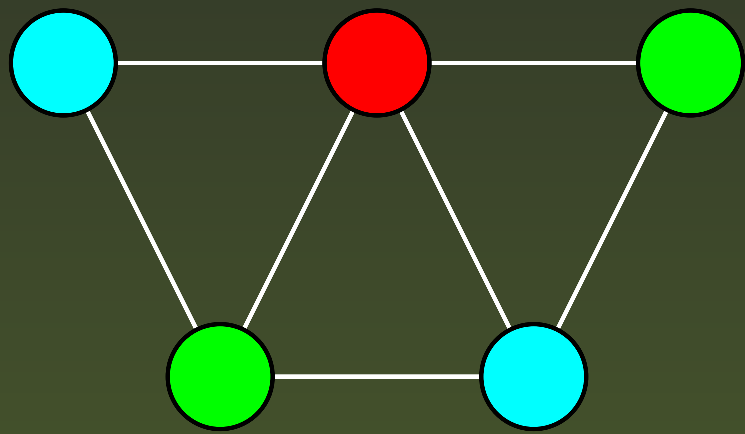
Ralph McKenzie found a partial solution of this problem which was then refined by Marcel Jackson. The solution will appear in their joint paper in “International Journal of Algebra and Computation”.

McKenzie–Jackson Theorem

Let C_3 be the graph used as an example 22 slides ago:



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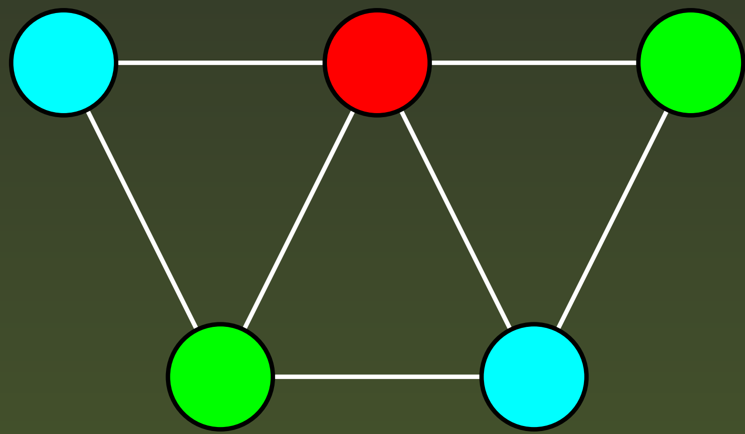

McKenzie and Jackson assign to each finite graph G a finite semigroup $S(G)$ such that $S(G) \in \operatorname{var} S(C_3)$ iff G belongs to the universal Horn class generated by C_3.

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If the graph G has n vertices, than the semigroup $S(G)$ has $n^2 + 5n + 5$ elements. In particular, the semigroup $S(C_3)$ has 55 elements. The structure of the semigroup $S(G)$ is rather transparent: it is a *local semilattice*.

Application to the Finite Basis Problem

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Comparing this with the McKenzie–Jackson Theorem, we conclude that the 55-element semigroup $S(C_3)$ is non-finitely based. This does not follow from any known result on the finite basis problem for finite semigroups!

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- constraints are determined by the edges of the graph: variables corresponding to adjacent vertices should be assigned different values.

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For instance, 3-COLOR can be easily identified with $\mathrm{CSP}(\Gamma)$ where the domain D is {red, green, blue} and Γ consists of the inequality relation $\neq_D$ on D. In particular, this shows that generally speaking $\mathrm{CSP}(\Gamma)$ is **NP**-complete.

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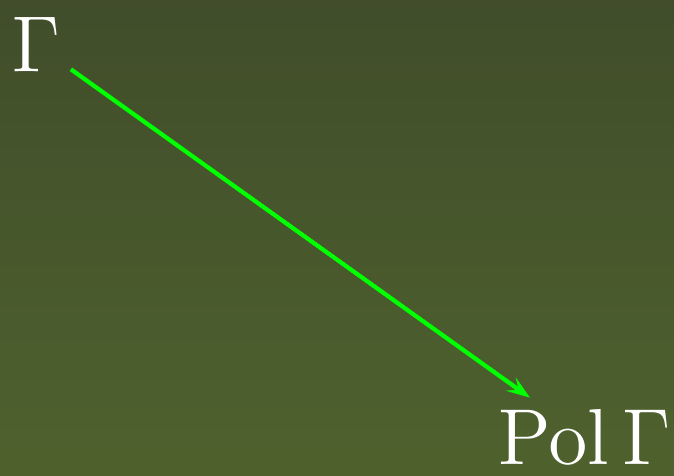

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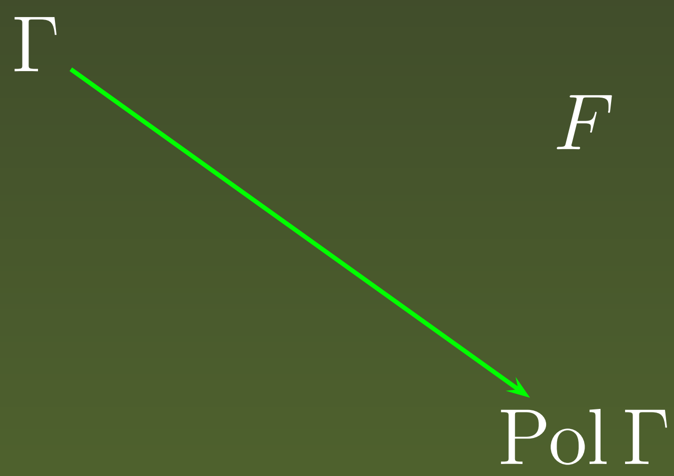

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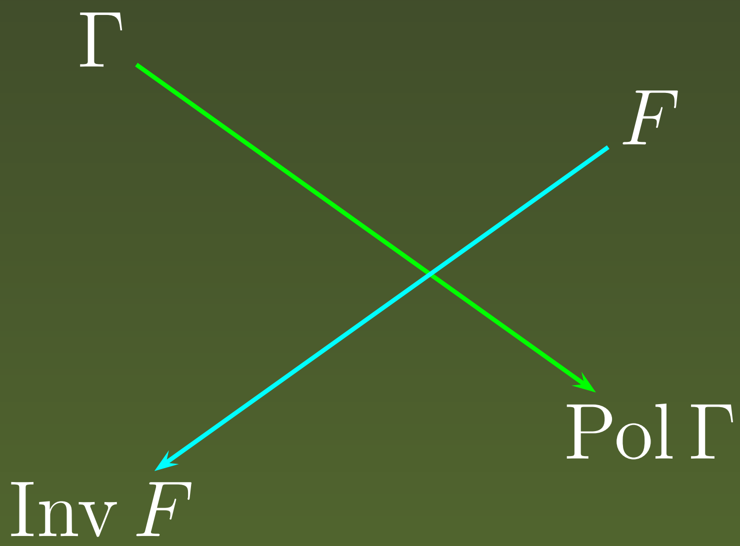

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It turns out that the complexity of $\mathrm{CSP}(\mathbf{D})$ depends on some deep algebraic properties of $\mathbf{D}$, in particular, on the Tame Congruence Theory labeling of its congruence lattice.

CSPs Defined by Semigroups

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$$ef = e^2 = e \;\&\; fe = f^2 = f \Rightarrow e = f,$$

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- the talk by Ondřej Klima on Complexity issues of checking identities in finite monoids (A317, $10^{45} - 11^{00}$) — it belongs to the Complexity of Algebra direction;
- the talk by László Zadori on Bounded width algebras (B321, $15^{05} - 15^{20}$) — it belongs to the Algebra of Complexity direction.

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I am looking forward to seeing all of you, dear friends, in Ekaterinburg next year!

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