The Finite Basis Problem for Kauffman Monoids

Mikhail Volkov
(joint with Karl Auinger, Yuzhu Chen, Xun Hu, and Yanfeng Luo)

Ural Federal University, Ekaterinburg, Russia

Plotkin-90, March 23rd, 2016

Outline

- Identities
- The Finite Basis Problem
- Wire monoids and Kauffman monoids
- Identities in Kauffman monoids

Outline

- Identities
- The Finite Basis Problem
- Wire monoids and Kauffman monoids
- Identities in Kauffman monoids

Outline

- Identities
- The Finite Basis Problem
- Wire monoids and Kauffman monoids
- Identities in Kauffman monoids

Outline

- Identities
- The Finite Basis Problem
- Wire monoids and Kauffman monoids
- Identities in Kauffman monoids

Identities

The idea of an identity or a law is very basic and is arguably one of the very first abstract ideas that school children encounter when they start to learn math.

I mean laws like the commutative law of addition:
A sum isn't changed at rearrangement of its addends.

At the end of the high school, a student is aware (or, at least, is supposed to be aware) of a good dozen of laws:
– the commutative and associative laws of addition,
– the commutative and associative laws of multiplication,
– the distributive law of multiplication over addition,
– the difference of two squares identity,
– the Pythagorean trigonometric identity,
etc, etc.

Identities

The idea of an identity or a law is very basic and is arguably one of the very first abstract ideas that school children encounter when they start to learn math.

I mean laws like the commutative law of addition:
A sum isn't changed at rearrangement of its addends.

At the end of the high school, a student is aware (or, at least, is supposed to be aware) of a good dozen of laws:
– the commutative and associative laws of addition,
– the commutative and associative laws of multiplication,
– the distributive law of multiplication over addition,
– the difference of two squares identity,
– the Pythagorean trigonometric identity,
etc, etc.

Identities

The idea of an identity or a law is very basic and is arguably one of the very first abstract ideas that school children encounter when they start to learn math.

I mean laws like the commutative law of addition:
A sum isn't changed at rearrangement of its addends.

At the end of the high school, a student is aware (or, at least, is supposed to be aware) of a good dozen of laws:
– the commutative and associative laws of addition,
– the commutative and associative laws of multiplication,
– the distributive law of multiplication over addition,
– the difference of two squares identity,
– the Pythagorean trigonometric identity,
etc, etc.

Identities

The idea of an identity or a law is very basic and is arguably one of the very first abstract ideas that school children encounter when they start to learn math.

I mean laws like the commutative law of addition:
A sum isn't changed at rearrangement of its addends.

At the end of the high school, a student is aware (or, at least, is supposed to be aware) of a good dozen of laws:
– the commutative and associative laws of addition,
– the commutative and associative laws of multiplication,
– the distributive law of multiplication over addition,
– the difference of two squares identity,
– the Pythagorean trigonometric identity,
etc, etc.

Inference of Identities

Moreover, the student may feel (though probably cannot explain) the difference between "main" or "primary" identities such as

$$ab = ba \qquad \text{(Comm-M)}$$

or

$$(ab)c = a(bc) \qquad \text{(Asso-M)}$$

and "secondary" ones such as, for instance,

$$(ab)^2 = a^2b^2. \qquad \text{(Example)}$$

"Primary" laws such as (Comm-M) or (Asso-M) are intrinsic properties of objects (say, numbers) we multiply and of the way the multiplication is defined while "secondary" identities can be formally inferred from "primary" ones without any knowledge of which objects are multiplied and how we define the multiplication.

Inference of Identities

Moreover, the student may feel (though probably cannot explain) the difference between "main" or "primary" identities such as

$$ab = ba \qquad \text{(Comm-M)}$$

or

$$(ab)c = a(bc) \qquad \text{(Asso-M)}$$

and "secondary" ones such as, for instance,

$$(ab)^2 = a^2b^2. \qquad \text{(Example)}$$

"Primary" laws such as (Comm-M) or (Asso-M) are intrinsic properties of objects (say, numbers) we multiply and of the way the multiplication is defined while "secondary" identities can be formally inferred from "primary" ones without any knowledge of which objects are multiplied and how we define the multiplication.

Inference of Identities

Moreover, the student may feel (though probably cannot explain) the difference between "main" or "primary" identities such as

$$ab = ba \qquad \text{(Comm-M)}$$

or

$$(ab)c = a(bc) \qquad \text{(Asso-M)}$$

and "secondary" ones such as, for instance,

$$(ab)^2 = a^2b^2. \qquad \text{(Example)}$$

"Primary" laws such as (Comm-M) or (Asso-M) are intrinsic properties of objects (say, numbers) we multiply and of the way the multiplication is defined while "secondary" identities can be formally inferred from "primary" ones without any knowledge of which objects are multiplied and how we define the multiplication.

Inference: Example

Here is a simple example of such a formal inference:

$$(ab)^2$$

Inference: Example

Here is a simple example of such a formal inference:

$$(ab)^2 = (ab)(ab) \qquad \text{by the definition of squaring}$$

Inference: Example

Here is a simple example of such a formal inference:

$$\begin{aligned}(ab)^2 &= (ab)(ab) && \text{by the definition of squaring}\\ &= a(ba)b && \text{by the law (Asso-M)}\end{aligned}$$

Inference: Example

Here is a simple example of such a formal inference:

$$\begin{aligned} (ab)^2 &= (ab)(ab) && \text{by the definition of squaring} \\ &= a(ba)b && \text{by the law (Asso-M)} \\ &= a(ab)b && \text{by the law (Comm-M)} \end{aligned}$$

Inference: Example

Here is a simple example of such a formal inference:

$$
\begin{aligned}
(ab)^2 &= (ab)(ab) && \text{by the definition of squaring}\\
&= a(ba)b && \text{by the law (Asso-M)}\\
&= a(ab)b && \text{by the law (Comm-M)}\\
&= (aa)(bb) && \text{by the law (Asso-M)}
\end{aligned}
$$

Inference: Example

Here is a simple example of such a formal inference:

$$
\begin{aligned}
(ab)^2 &= (ab)(ab) && \text{by the definition of squaring}\\
&= a(ba)b && \text{by the law (Asso-M)}\\
&= a(ab)b && \text{by the law (Comm-M)}\\
&= (aa)(bb) && \text{by the law (Asso-M)}\\
&= a^2b^2 && \text{by the definition of squaring}
\end{aligned}
$$

Inference: Example

Here is a simple example of such a formal inference:

$$
\begin{aligned}
(ab)^2 &= (ab)(ab) && \text{by the definition of squaring}\\
&= a(ba)b && \text{by the law (Asso-M)}\\
&= a(ab)b && \text{by the law (Comm-M)}\\
&= (aa)(bb) && \text{by the law (Asso-M)}\\
&= a^2b^2 && \text{by the definition of squaring}
\end{aligned}
$$

Thus, (Example) is a formal corollary of (Asso-M) and (Comm-M) and holds whenever and wherever the two laws hold.

Inference: Example

Here is a simple example of such a formal inference:

$$\begin{aligned}
(ab)^2 &= (ab)(ab) && \text{by the definition of squaring}\\
&= a(ba)b && \text{by the law (Asso-M)}\\
&= a(ab)b && \text{by the law (Comm-M)}\\
&= (aa)(bb) && \text{by the law (Asso-M)}\\
&= a^2b^2 && \text{by the definition of squaring}
\end{aligned}$$

Thus, (Example) is a formal corollary of (Asso-M) and (Comm-M) and holds whenever and wherever the two laws hold.

That's why, when extending $\mathbb{N}$ to $\mathbb{Z}$, and then to $\mathbb{Q}$, and then to $\mathbb{R}$, and then to $\mathbb{C}$, we have to care of preserving (Asso-M) and (Comm-M) but there is no need to care of preserving (Example).

Identity Basis

A big part of algebra in fact deals with inferring some useful "secondary identities" from some "primary" laws. Identities to be inferred may be quite complicated, and the inference itself may be highly non-trivial—think, for instance, of the product rule for determinant: $\det AB = \det A \det B$.
However, one can observe that usually only a few 'primary' laws are invoked in the course of inference.

This observation leads to the idea of composing a complete list of 'primary' laws that would allow us to infer every possible identity. Such a list is called an identity basis.

Warning: the word 'basis' here doesn't mean any independence assumption! Hence no uniqueness, etc.

Identity Basis

A big part of algebra in fact deals with inferring some useful "secondary identities" from some "primary" laws. Identities to be inferred may be quite complicated, and the inference itself may be highly non-trivial—think, for instance, of the product rule for determinant: $\det AB = \det A \det B$.

However, one can observe that usually only a few 'primary' laws are invoked in the course of inference.

This observation leads to the idea of composing a complete list of 'primary' laws that would allow us to infer every possible identity. Such a list is called an identity basis.

Warning: the word 'basis' here doesn't mean any independence assumption! Hence no uniqueness, etc.

Identity Basis

A big part of algebra in fact deals with inferring some useful "secondary identities" from some "primary" laws. Identities to be inferred may be quite complicated, and the inference itself may be highly non-trivial—think, for instance, of the product rule for determinant: $\det AB = \det A \det B$.

However, one can observe that usually only a few 'primary' laws are invoked in the course of inference.

This observation leads to the idea of composing a complete list of 'primary' laws that would allow us to infer every possible identity. Such a list is called an identity basis.

Warning: the word 'basis' here doesn't mean any independence assumption! Hence no uniqueness, etc.

Identity Basis

A big part of algebra in fact deals with inferring some useful "secondary identities" from some "primary" laws. Identities to be inferred may be quite complicated, and the inference itself may be highly non-trivial—think, for instance, of the product rule for determinant: $\det AB = \det A \det B$.
However, one can observe that usually only a few 'primary' laws are invoked in the course of inference.

This observation leads to the idea of composing a complete list of 'primary' laws that would allow us to infer every possible identity. Such a list is called an identity basis.

Warning: the word 'basis' here doesn't mean any independence assumption! Hence no uniqueness, etc.

Identity Basis

A big part of algebra in fact deals with inferring some useful "secondary identities" from some "primary" laws. Identities to be inferred may be quite complicated, and the inference itself may be highly non-trivial—think, for instance, of the product rule for determinant: $\det AB = \det A \det B$.
However, one can observe that usually only a few 'primary' laws are invoked in the course of inference.

This observation leads to the idea of composing a complete list of 'primary' laws that would allow us to infer every possible identity. Such a list is called an identity basis.

Warning: the word 'basis' here doesn't mean any independence assumption! Hence no uniqueness, etc.

Identity Basis

A big part of algebra in fact deals with inferring some useful "secondary identities" from some "primary" laws. Identities to be inferred may be quite complicated, and the inference itself may be highly non-trivial—think, for instance, of the product rule for determinant: $\det AB = \det A \det B$.
However, one can observe that usually only a few 'primary' laws are invoked in the course of inference.

This observation leads to the idea of composing a complete list of 'primary' laws that would allow us to infer every possible identity. Such a list is called an identity basis.

Warning: the word 'basis' here doesn't mean any independence assumption! Hence no uniqueness, etc.

High School Identities-I

Of course, in order to speak about an identity basis, one has to specify which identities are under consideration. More precisely, one has to specify 1) a set of objects (say, numbers, or functions, or matrices, etc) and 2) a set of operations on these objects (say, addition, and/or multiplication, and/or exponentiation, etc).

For instance, let our objects be natural numbers (i.e. positive integers) and let our operations be addition and multiplication. Then it is not hard to show that the following 6 laws form a basis:

$$a + b = b + a,$$

$$a + (b + c) = (a + b) + c,$$

$$a \cdot 1 = a,$$

$$a \cdot b = b \cdot a,$$

$$a \cdot (b \cdot c) = (a \cdot b) \cdot c,$$

$$a \cdot (b + c) = a \cdot b + a \cdot c.$$

High School Identities-I

Of course, in order to speak about an identity basis, one has to specify which identities are under consideration. More precisely, one has to specify 1) a set of objects (say, numbers, or functions, or matrices, etc) and 2) a set of operations on these objects (say, addition, and/or multiplication, and/or exponentiation, etc).

For instance, let our objects be natural numbers (i.e. positive integers) and let our operations be addition and multiplication. Then it is not hard to show that the following 6 laws form a basis:

$$a + b = b + a,$$
$$a + (b + c) = (a + b) + c,$$
$$a \cdot 1 = a,$$
$$a \cdot b = b \cdot a,$$
$$a \cdot (b \cdot c) = (a \cdot b) \cdot c,$$
$$a \cdot (b + c) = a \cdot b + a \cdot c.$$

High School Identities-I

Of course, in order to speak about an identity basis, one has to specify which identities are under consideration. More precisely, one has to specify 1) a set of objects (say, numbers, or functions, or matrices, etc) and 2) a set of operations on these objects (say, addition, and/or multiplication, and/or exponentiation, etc).

For instance, let our objects be natural numbers (i.e. positive integers) and let our operations be addition and multiplication.

Then it is not hard to show that the following 6 laws form a basis:

$$a + b = b + a,$$

$$a + (b + c) = (a + b) + c,$$

$$a \cdot 1 = a,$$

$$a \cdot b = b \cdot a,$$

$$a \cdot (b \cdot c) = (a \cdot b) \cdot c,$$

$$a \cdot (b + c) = a \cdot b + a \cdot c.$$

High School Identities-I

Of course, in order to speak about an identity basis, one has to specify which identities are under consideration. More precisely, one has to specify 1) a set of objects (say, numbers, or functions, or matrices, etc) and 2) a set of operations on these objects (say, addition, and/or multiplication, and/or exponentiation, etc).

For instance, let our objects be natural numbers (i.e. positive integers) and let our operations be addition and multiplication. Then it is not hard to show that the following 6 laws form a basis:

$$a + b = b + a,$$
$$a + (b + c) = (a + b) + c,$$
$$a \cdot 1 = a,$$
$$a \cdot b = b \cdot a,$$
$$a \cdot (b \cdot c) = (a \cdot b) \cdot c,$$
$$a \cdot (b + c) = a \cdot b + a \cdot c.$$

High School Identities-II

Another "high school" operation on the set $\mathbb{N}$ is exponentiation.
High school students know the following 5 laws involving addition, multiplication, and exponentiation:

$$1^a = 1,$$
$$a^1 = a,$$
$$a^{b+c} = a^b \cdot a^c,$$
$$(a \cdot b)^c = a^c \cdot b^c,$$
$$(a^b)^c = a^{b \cdot c}.$$

We collectively refer to the 11 "standard" laws (the 6 from the previous slide and the 5 from this slide) as (HSI).

High School Identities-II

Another "high school" operation on the set $\mathbb{N}$ is exponentiation. High school students know the following 5 laws involving addition, multiplication, and exponentiation:

$$1^a = 1,$$
$$a^1 = a,$$
$$a^{b+c} = a^b \cdot a^c,$$
$$(a \cdot b)^c = a^c \cdot b^c,$$
$$(a^b)^c = a^{b \cdot c}.$$

We collectively refer to the 11 "standard" laws (the 6 from the previous slide and the 5 from this slide) as (HSI).

High School Identities-II

Another "high school" operation on the set $\mathbb{N}$ is exponentiation. High school students know the following 5 laws involving addition, multiplication, and exponentiation:

$$1^a = 1,$$
$$a^1 = a,$$
$$a^{b+c} = a^b \cdot a^c,$$
$$(a \cdot b)^c = a^c \cdot b^c,$$
$$(a^b)^c = a^{b \cdot c}.$$

We collectively refer to the 11 "standard" laws (the 6 from the previous slide and the 5 from this slide) as (HSI).

Arguably, it was Richard Dedekind who (in his famous book "Was sind und was sollen die Zahlen?" of 1888) seemed to be asking if the 11 laws (HSI) were somehow sufficient to tell us everything we could want to know about the natural numbers.

At that time, however, no mathematical language existed in which such a question could be stated precisely.

Such a language was developed in the first half of the 20th century, and Alfred Tarski was one of the major contributor to this development. In the 1960s Tarski formulated the problem in the terms that we use nowadays:

Arguably, it was Richard Dedekind who (in his famous book "Was sind und was sollen die Zahlen?" of 1888) seemed to be asking if the 11 laws (HSI) were somehow sufficient to tell us everything we could want to know about the natural numbers.

At that time, however, no mathematical language existed in which such a question could be stated precisely.

Such a language was developed in the first half of the 20th century, and Alfred Tarski was one of the major contributor to this development. In the 1960s Tarski formulated the problem in the terms that we use nowadays:

Tarski's HSI Problem

Arguably, it was Richard Dedekind who (in his famous book "Was sind und was sollen die Zahlen?" of 1888) seemed to be asking if the 11 laws (HSI) were somehow sufficient to tell us everything we could want to know about the natural numbers.
At that time, however, no mathematical language existed in which such a question could be stated precisely.
Such a language was developed in the first half of the 20th century, and Alfred Tarski was one of the major contributor to this development. In the 1960s Tarski formulated the problem in the terms that we use nowadays:

Tarski's HSI Problem

Arguably, it was Richard Dedekind who (in his famous book "Was sind und was sollen die Zahlen?" of 1888) seemed to be asking if the 11 laws (HSI) were somehow sufficient to tell us everything we could want to know about the natural numbers.

At that time, however, no mathematical language existed in which such a question could be stated precisely.

Such a language was developed in the first half of the 20th century, and Alfred Tarski was one of the major contributor to this development. In the 1960s Tarski formulated the problem in the terms that we use nowadays:

Tarski's HSI Problem

Do the laws (HSI) form a basis for the identities that involve addition, multiplication, and exponentiation and hold in $\mathbb{N}$?

Tarski's HSI Problem

Arguably, it was Richard Dedekind who (in his famous book "Was sind und was sollen die Zahlen?" of 1888) seemed to be asking if the 11 laws (HSI) were somehow sufficient to tell us everything we could want to know about the natural numbers.
At that time, however, no mathematical language existed in which such a question could be stated precisely.
Such a language was developed in the first half of the 20th century, and Alfred Tarski was one of the major contributor to this development. In the 1960s Tarski formulated the problem in the terms that we use nowadays:

Tarski's HSI Problem

Do the laws (HSI) form a basis for the identities that involve addition, multiplication, and exponentiation and hold in $\mathbb{N}$?

Surprisingly, the answer is NO.

Wilkie's Identity

In 1980 Alex Wilkie found the following identity that holds in $\mathbb{N}$ but cannot be inferred from (HSI).

$$\left((1+a)^a+(1+a+a^2)^a\right)^b\cdot\left((1+a^3)^b+(1+a^2+a^4)^b\right)^a=$$
$$=\left((1+a)^b+(1+a+a^2)^b\right)^a\cdot\left((1+a^3)^a+(1+a^2+a^4)^a\right)^b.$$

Wilkie's identity looks complicated but in fact it is easy to show that it holds in $\mathbb{N}$.

A more delicate question is how to prove that the identity cannot be inferred from (HSI). For this, one construct a counter-model: a set M with 3 operations such that (HSI) hold in M but Wilkie's identity does not. A counter-model with 12 elements is known.

Wilkie's Identity

In 1980 Alex Wilkie found the following identity that holds in $\mathbb{N}$ but cannot be inferred from (HSI).

$$\left((1+a)^a + (1+a+a^2)^a\right)^b \cdot \left((1+a^3)^b + (1+a^2+a^4)^b\right)^a =$$
$$= \left((1+a)^b + (1+a+a^2)^b\right)^a \cdot \left((1+a^3)^a + (1+a^2+a^4)^a\right)^b.$$

Wilkie's identity looks complicated but in fact it is easy to show that it holds in $\mathbb{N}$.

A more delicate question is how to prove that the identity cannot be inferred from (HSI). For this, one construct a counter-model: a set M with 3 operations such that (HSI) hold in M but Wilkie's identity does not. A counter-model with 12 elements is known.

Wilkie's Identity

In 1980 Alex Wilkie found the following identity that holds in $\mathbb{N}$ but cannot be inferred from (HSI).

$$\left((1+a)^a + (1+a+a^2)^a\right)^b \cdot \left((1+a^3)^b + (1+a^2+a^4)^b\right)^a =$$
$$= \left((1+a)^b + (1+a+a^2)^b\right)^a \cdot \left((1+a^3)^a + (1+a^2+a^4)^a\right)^b.$$

Wilkie's identity looks complicated but in fact it is easy to show that it holds in $\mathbb{N}$.

A more delicate question is how to prove that the identity cannot be inferred from (HSI). For this, one construct a counter-model: a set M with 3 operations such that (HSI) hold in M but Wilkie's identity does not. A counter-model with 12 elements is known.

Wilkie's Identity

In 1980 Alex Wilkie found the following identity that holds in $\mathbb{N}$ but cannot be inferred from (HSI).

$$((1+a)^a + (1+a+a^2)^a)^b \cdot \left((1+a^3)^b + (1+a^2+a^4)^b\right)^a =$$
$$= \left((1+a)^b + (1+a+a^2)^b\right)^a \cdot ((1+a^3)^a + (1+a^2+a^4)^a)^b.$$

Wilkie's identity looks complicated but in fact it is easy to show that it holds in $\mathbb{N}$.

A more delicate question is how to prove that the identity cannot be inferred from (HSI). For this, one construct a counter-model: a set M with 3 operations such that (HSI) hold in M but Wilkie's identity does not. A counter-model with 12 elements is known.

Wilkie's Identity

In 1980 Alex Wilkie found the following identity that holds in $\mathbb{N}$ but cannot be inferred from (HSI).

$$((1+a)^a + (1+a+a^2)^a)^b \cdot \left((1+a^3)^b + (1+a^2+a^4)^b\right)^a =$$
$$= \left((1+a)^b + (1+a+a^2)^b\right)^a \cdot ((1+a^3)^a + (1+a^2+a^4)^a)^b.$$

Wilkie's identity looks complicated but in fact it is easy to show that it holds in $\mathbb{N}$.

A more delicate question is how to prove that the identity cannot be inferred from (HSI). For this, one construct a counter-model: a set M with 3 operations such that (HSI) hold in M but Wilkie's identity does not. A counter-model with 12 elements is known.

Wilkie's Identity

In 1980 Alex Wilkie found the following identity that holds in $\mathbb{N}$ but cannot be inferred from (HSI).

$$((1+a)^a + (1+a+a^2)^a)^b \cdot \left((1+a^3)^b + (1+a^2+a^4)^b\right)^a =$$
$$= \left((1+a)^b + (1+a+a^2)^b\right)^a \cdot ((1+a^3)^a + (1+a^2+a^4)^a)^b.$$

Wilkie's identity looks complicated but in fact it is easy to show that it holds in $\mathbb{N}$.

A more delicate question is how to prove that the identity cannot be inferred from (HSI). For this, one construct a counter-model: a set M with 3 operations such that (HSI) hold in M but Wilkie's identity does not. A counter-model with 12 elements is known.

No Finite Basis for $(\mathbb{N}; +, \cdot, \uparrow)$

Can one save the situation by including Wilkie's identity in the high school curriculum? Fortunately, for kids, this is not possible: the identities of $(\mathbb{N}; +, \cdot, \uparrow)$ admit no finite basis. (This was shown by R. Gurevič in "Equational theory of positive numbers with exponentiation is not finitely axiomatizable", Ann. Pure and Applied Logic 49 (1990) 1–30.) Thus, if one chooses any finite set Σ of identities of $(\mathbb{N}; +, \cdot, \uparrow)$, there always exists an identity τ that plays the same role with respect to Σ as Wilkie's identity does with respect to (HSI): τ holds in $\mathbb{N}$ but cannot be inferred from Σ.

Here we encounter the phenomenon when the identities of a natural and apparently simple structure admit no finite basis. In this situation, we say that the answer to the Finite Basis Problem (FBP) for the structure is negative and the structure is nonfinitely based. Otherwise it is finitely based. Thus, $(\mathbb{N}; +, \cdot)$ is finitely based while $(\mathbb{N}; +, \cdot, \uparrow)$ is not.

No Finite Basis for $(\mathbb{N}; +, \cdot, \uparrow)$

Can one save the situation by including Wilkie's identity in the high school curriculum? Fortunately, for kids, this is not possible: the identities of $(\mathbb{N}; +, \cdot, \uparrow)$ admit no finite basis. (This was shown by R. Gurevič in "Equational theory of positive numbers with exponentiation is not finitely axiomatizable", Ann. Pure and Applied Logic 49 (1990) 1–30.) Thus, if one chooses any finite set Σ of identities of $(\mathbb{N}; +, \cdot, \uparrow)$, there always exists an identity τ that plays the same role with respect to Σ as Wilkie's identity does with respect to (HSI): τ holds in $\mathbb{N}$ but cannot be inferred from Σ.

Here we encounter the phenomenon when the identities of a natural and apparently simple structure admit no finite basis. In this situation, we say that the answer to the Finite Basis Problem (FBP) for the structure is negative and the structure is nonfinitely based. Otherwise it is finitely based. Thus, $(\mathbb{N}; +, \cdot)$ is finitely based while $(\mathbb{N}; +, \cdot, \uparrow)$ is not.

No Finite Basis for $(\mathbb{N}; +, \cdot, \uparrow)$

Can one save the situation by including Wilkie's identity in the high school curriculum? Fortunately, for kids, this is not possible: the identities of $(\mathbb{N}; +, \cdot, \uparrow)$ admit no finite basis. (This was shown by R. Gurevič in "Equational theory of positive numbers with exponentiation is not finitely axiomatizable", Ann. Pure and Applied Logic 49 (1990) 1–30.) Thus, if one chooses any finite set Σ of identities of $(\mathbb{N}; +, \cdot, \uparrow)$, there always exists an identity τ that plays the same role with respect to Σ as Wilkie's identity does with respect to (HSI): τ holds in $\mathbb{N}$ but cannot be inferred from Σ.

Here we encounter the phenomenon when the identities of a natural and apparently simple structure admit no finite basis. In this situation, we say that the answer to the Finite Basis Problem (FBP) for the structure is negative and the structure is nonfinitely based. Otherwise it is finitely based. Thus, $(\mathbb{N}; +, \cdot)$ is finitely based while $(\mathbb{N}; +, \cdot, \uparrow)$ is not.

No Finite Basis for $(\mathbb{N}; +, \cdot, \uparrow)$

Can one save the situation by including Wilkie's identity in the high school curriculum? Fortunately, for kids, this is not possible: the identities of $(\mathbb{N}; +, \cdot, \uparrow)$ admit no finite basis. (This was shown by R. Gurevič in "Equational theory of positive numbers with exponentiation is not finitely axiomatizable", Ann. Pure and Applied Logic 49 (1990) 1–30.) Thus, if one chooses any finite set Σ of identities of $(\mathbb{N}; +, \cdot, \uparrow)$, there always exists an identity τ that plays the same role with respect to Σ as Wilkie's identity does with respect to (HSI): τ holds in $\mathbb{N}$ but cannot be inferred from Σ.

Here we encounter the phenomenon when the identities of a natural and apparently simple structure admit no finite basis. In this situation, we say that the answer to the Finite Basis Problem (FBP) for the structure is negative and the structure is nonfinitely based. Otherwise it is finitely based. Thus, $(\mathbb{N}; +, \cdot)$ is finitely based while $(\mathbb{N}; +, \cdot, \uparrow)$ is not.

No Finite Basis for $(\mathbb{N}; +, \cdot, \uparrow)$

Can one save the situation by including Wilkie's identity in the high school curriculum? Fortunately, for kids, this is not possible: the identities of $(\mathbb{N}; +, \cdot, \uparrow)$ admit no finite basis. (This was shown by R. Gurevič in "Equational theory of positive numbers with exponentiation is not finitely axiomatizable", Ann. Pure and Applied Logic 49 (1990) 1–30.) Thus, if one chooses any finite set Σ of identities of $(\mathbb{N}; +, \cdot, \uparrow)$, there always exists an identity τ that plays the same role with respect to Σ as Wilkie's identity does with respect to (HSI): τ holds in $\mathbb{N}$ but cannot be inferred from Σ.

Here we encounter the phenomenon when the identities of a natural and apparently simple structure admit no finite basis. In this situation, we say that the answer to the Finite Basis Problem (FBP) for the structure is negative and the structure is nonfinitely based. Otherwise it is finitely based.

Thus, $(\mathbb{N}; +, \cdot)$ is finitely based while $(\mathbb{N}; +, \cdot, \uparrow)$ is not.

No Finite Basis for $(\mathbb{N}; +, \cdot, \uparrow)$

Can one save the situation by including Wilkie's identity in the high school curriculum? Fortunately, for kids, this is not possible: the identities of $(\mathbb{N}; +, \cdot, \uparrow)$ admit no finite basis. (This was shown by R. Gurevič in "Equational theory of positive numbers with exponentiation is not finitely axiomatizable", Ann. Pure and Applied Logic 49 (1990) 1–30.) Thus, if one chooses any finite set Σ of identities of $(\mathbb{N}; +, \cdot, \uparrow)$, there always exists an identity τ that plays the same role with respect to Σ as Wilkie's identity does with respect to (HSI): τ holds in $\mathbb{N}$ but cannot be inferred from Σ.

Here we encounter the phenomenon when the identities of a natural and apparently simple structure admit no finite basis.

In this situation, we say that the answer to the Finite Basis Problem (FBP) for the structure is negative and the structure is nonfinitely based. Otherwise it is finitely based.

Thus, $(\mathbb{N}; +, \cdot)$ is finitely based while $(\mathbb{N}; +, \cdot, \uparrow)$ is not.

No Finite Basis for $(\mathbb{N}; +, \cdot, \uparrow)$

Can one save the situation by including Wilkie's identity in the high school curriculum? Fortunately, for kids, this is not possible: the identities of $(\mathbb{N}; +, \cdot, \uparrow)$ admit no finite basis. (This was shown by R. Gurevič in "Equational theory of positive numbers with exponentiation is not finitely axiomatizable", Ann. Pure and Applied Logic 49 (1990) 1–30.) Thus, if one chooses any finite set Σ of identities of $(\mathbb{N}; +, \cdot, \uparrow)$, there always exists an identity τ that plays the same role with respect to Σ as Wilkie's identity does with respect to (HSI): τ holds in $\mathbb{N}$ but cannot be inferred from Σ.

Here we encounter the phenomenon when the identities of a natural and apparently simple structure admit no finite basis. In this situation, we say that the answer to the Finite Basis Problem (FBP) for the structure is negative and the structure is nonfinitely based. Otherwise it is finitely based. Thus, $(\mathbb{N}; +, \cdot)$ is finitely based while $(\mathbb{N}; +, \cdot, \uparrow)$ is not.

No Finite Basis for $(\mathbb{N}; +, \cdot, \uparrow)$

Can one save the situation by including Wilkie's identity in the high school curriculum? Fortunately, for kids, this is not possible: the identities of $(\mathbb{N}; +, \cdot, \uparrow)$ admit no finite basis. (This was shown by R. Gurevič in "Equational theory of positive numbers with exponentiation is not finitely axiomatizable", Ann. Pure and Applied Logic 49 (1990) 1–30.) Thus, if one chooses any finite set Σ of identities of $(\mathbb{N}; +, \cdot, \uparrow)$, there always exists an identity τ that plays the same role with respect to Σ as Wilkie's identity does with respect to (HSI): τ holds in $\mathbb{N}$ but cannot be inferred from Σ.

Here we encounter the phenomenon when the identities of a natural and apparently simple structure admit no finite basis. In this situation, we say that the answer to the Finite Basis Problem (FBP) for the structure is negative and the structure is nonfinitely based. Otherwise it is finitely based.

Thus, $(\mathbb{N}; +, \cdot)$ is finitely based while $(\mathbb{N}; +, \cdot, \uparrow)$ is not.

No Finite Basis for $(\mathbb{N}; +, \cdot, \uparrow)$

Can one save the situation by including Wilkie's identity in the high school curriculum? Fortunately, for kids, this is not possible: the identities of $(\mathbb{N}; +, \cdot, \uparrow)$ admit no finite basis. (This was shown by R. Gurevič in "Equational theory of positive numbers with exponentiation is not finitely axiomatizable", Ann. Pure and Applied Logic 49 (1990) 1–30.) Thus, if one chooses any finite set Σ of identities of $(\mathbb{N}; +, \cdot, \uparrow)$, there always exists an identity τ that plays the same role with respect to Σ as Wilkie's identity does with respect to (HSI): τ holds in $\mathbb{N}$ but cannot be inferred from Σ.

Here we encounter the phenomenon when the identities of a natural and apparently simple structure admit no finite basis. In this situation, we say that the answer to the Finite Basis Problem (FBP) for the structure is negative and the structure is nonfinitely based. Otherwise it is finitely based. Thus, $(\mathbb{N}; +, \cdot)$ is finitely based while $(\mathbb{N}; +, \cdot, \uparrow)$ is not.

It is the FBP that underlies the research reported in this talk.

The FBP is natural by itself, but it has also revealed a number of interesting and unexpected relations to many issues of theoretical and practical importance ranging from feasible algorithms for membership in certain classes of formal languages to classical number-theoretic conjectures such as the Twin Prime, Goldbach, existence of odd perfect numbers and the infinitude of even perfect numbers. (See P. Perkins, "Finite axiomatizability for equational theories of computable groupoids", J. Symbolic Logic 54 (1989), 1018–1022, where it is shown that each of these conjectures is equivalent to the FBP for a structure of the form $(S, \cdot)$.)

It is the FBP that underlies the research reported in this talk.

The Finite Basis Problem

Given an interesting structure M (a set with a bunch of operations on it), determine whether of not M is finitely based.

The FBP is natural by itself, but it has also revealed a number of interesting and unexpected relations to many issues of theoretical and practical importance ranging from feasible algorithms for membership in certain classes of formal languages to classical number-theoretic conjectures such as the Twin Prime, Goldbach, existence of odd perfect numbers and the infinitude of even perfect numbers. (See P. Perkins, "Finite axiomatizability for equational theories of computable groupoids", J. Symbolic Logic 54 (1989), 1018–1022, where it is shown that each of these conjectures is equivalent to the FBP for a structure of the form $(S, \cdot)$.)

The Finite Basis Problem

It is the FBP that underlies the research reported in this talk.

The Finite Basis Problem

Given an interesting structure M (a set with a bunch of operations on it), determine whether of not M is finitely based.

The FBP is natural by itself, but it has also revealed a number of interesting and unexpected relations to many issues of theoretical and practical importance ranging from feasible algorithms for membership in certain classes of formal languages to classical number-theoretic conjectures such as the Twin Prime, Goldbach, existence of odd perfect numbers and the infinitude of even perfect numbers. (See P. Perkins, “Finite axiomatizability for equational theories of computable groupoids”, J. Symbolic Logic 54 (1989), 1018–1022, where it is shown that each of these conjectures is equivalent to the FBP for a structure of the form $(S, \cdot)$.)

It is the FBP that underlies the research reported in this talk.

The Finite Basis Problem

Given an interesting structure M (a set with a bunch of operations on it), determine whether of not M is finitely based.

The FBP is natural by itself, but it has also revealed a number of interesting and unexpected relations to many issues of theoretical and practical importance ranging from feasible algorithms for membership in certain classes of formal languages to classical number-theoretic conjectures such as the Twin Prime, Goldbach, existence of odd perfect numbers and the infinitude of even perfect numbers. (See P. Perkins, "Finite axiomatizability for equational theories of computable groupoids", J. Symbolic Logic 54 (1989), 1018–1022, where it is shown that each of these conjectures is equivalent to the FBP for a structure of the form $(S, \cdot)$.)

The Finite Basis Problem for Finite Structures

Even a finite structure can be nonfinitely based. The smallest example is a 3-element structure of the form $(S, \cdot)$ known as Murskiĭ's groupoid, but, IMHO, the most striking example (the Brandt monoid) is formed by the following six 2×2-matrices:

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix},$$

the operation being the usual matrix multiplication. (This example is due to P. Perkins, "Bases for equational theories of semigroups", J. Algebra 11 (1969) 298–314.)

Thus, here we see a very transparent, very natural, and very finite structure whose identities cannot be classified by finite means.

Even a finite structure can be nonfinitely based. The smallest example is a 3-element structure of the form $(S, \cdot)$ known as Murskiĭ's groupoid, but, IMHO, the most striking example (the Brandt monoid) is formed by the following six 2×2-matrices:

$$\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}, \begin{pmatrix}1 & 0\\ 0 & 0\end{pmatrix}, \begin{pmatrix}0 & 1\\ 0 & 0\end{pmatrix}, \begin{pmatrix}0 & 0\\ 1 & 0\end{pmatrix}, \begin{pmatrix}0 & 0\\ 0 & 1\end{pmatrix}, \begin{pmatrix}0 & 0\\ 0 & 0\end{pmatrix},$$

the operation being the usual matrix multiplication. (This example is due to P. Perkins, "Bases for equational theories of semigroups", J. Algebra 11 (1969) 298–314.)

Thus, here we see a very transparent, very natural, and very finite structure whose identities cannot be classified by finite means.

Even a finite structure can be nonfinitely based. The smallest example is a 3-element structure of the form $(S, \cdot)$ known as Murskiĭ's groupoid, but, IMHO, the most striking example (the Brandt monoid) is formed by the following six 2×2-matrices:

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix},$$

the operation being the usual matrix multiplication. (This example is due to P. Perkins, "Bases for equational theories of semigroups", J. Algebra 11 (1969) 298–314.)

Thus, here we see a very transparent, very natural, and very finite structure whose identities cannot be classified by finite means.

Even a finite structure can be nonfinitely based. The smallest example is a 3-element structure of the form $(S, \cdot)$ known as Murskiĭ's groupoid, but, IMHO, the most striking example (the Brandt monoid) is formed by the following six 2×2-matrices:

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix},$$

the operation being the usual matrix multiplication. (This example is due to P. Perkins, "Bases for equational theories of semigroups", J. Algebra 11 (1969) 298–314.)

Thus, here we see a very transparent, very natural, and very finite structure whose identities cannot be classified by finite means.

Even a finite structure can be nonfinitely based. The smallest example is a 3-element structure of the form $(S, \cdot)$ known as Murskiĭ's groupoid, but, IMHO, the most striking example (the Brandt monoid) is formed by the following six 2×2-matrices:

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix},$$

the operation being the usual matrix multiplication. (This example is due to P. Perkins, "Bases for equational theories of semigroups", J. Algebra 11 (1969) 298–314.)

Thus, here we see a very transparent, very natural, and very finite structure whose identities cannot be classified by finite means.

Tarski's Finite Basis Problem

In the early 1960's, Tarski suggested to study the FBP for finite structures as a decision problem. Indeed, since any finite structure S is an object that can be given in a constructive way, one can ask for an algorithm which when presented with an effective description of S, would determine whether or not S is finitely based.

In the early 1960's, Tarski suggested to study the FBP for finite structures as a decision problem. Indeed, since any finite structure S is an object that can be given in a constructive way, one can ask for an algorithm which when presented with an effective description of S, would determine whether or not S is finitely based.

In the early 1960's, Tarski suggested to study the FBP for finite structures as a decision problem. Indeed, since any finite structure S is an object that can be given in a constructive way, one can ask for an algorithm which when presented with an effective description of S, would determine whether or not S is finitely based.

Tarski's Finite Basis Problem

Is there an algorithm that when given an effective description of a finite structure S decides whether S is finitely based or not?

Tarski's Finite Basis Problem

In the early 1960's, Tarski suggested to study the FBP for finite structures as a decision problem. Indeed, since any finite structure S is an object that can be given in a constructive way, one can ask for an algorithm which when presented with an effective description of S, would determine whether or not S is finitely based.

Tarski's Finite Basis Problem

Is there an algorithm that when given an effective description of a finite structure S decides whether S is finitely based or not?

This fundamental question was answered in the negative by Ralph McKenzie ("Tarski's finite basis problem is undecidable", Int. J. Algebra and Computation 6 (1996) 49–104), even for finite structures with a single operation!

In the early 1960's, Tarski suggested to study the FBP for finite structures as a decision problem. Indeed, since any finite structure S is an object that can be given in a constructive way, one can ask for an algorithm which when presented with an effective description of S, would determine whether or not S is finitely based.

Tarski's Finite Basis Problem

Is there an algorithm that when given an effective description of a finite structure S decides whether S is finitely based or not?

This fundamental question was answered in the negative by Ralph McKenzie ("Tarski's finite basis problem is undecidable", Int. J. Algebra and Computation 6 (1996) 49–104), even for finite structures with a single operation!

I think it is a good news for people involved in studying the FBP: since no mechanical procedure exists, you should be clever than your computer to get an answer!

Advertisement

Semigroups and monoids are the only "classical" algebras for which finite nonfinitely based objects exist: finite groups, associative or Lie rings, lattices are finitely based. Tarski's problem for finite semigroups and monoids still remains open.

There is a survey paper on the FBP for finite semigroups:
M.V. Volkov, The finite basis problem for finite semigroups, *Sci. Math. Jap.*, Vol. 53, 171–199, 2001.

The area has been progressed a lot over the last 15 years.

A (partially) updated version of the above survey is available through my webpage:
`http://csseminar.kadm.usu.ru/volkov`

Edmond W. H. Lee and I are working on a new survey (to be submitted this summer).

Semigroups and monoids are the only “classical” algebras for which finite nonfinitely based objects exist: finite groups, associative or Lie rings, lattices are finitely based. Tarski’s problem for finite semigroups and monoids still remains open.

There is a survey paper on the FBP for finite semigroups:
M.V. Volkov, The finite basis problem for finite semigroups, *Sci. Math. Jap.*, Vol. 53, 171–199, 2001.

The area has been progressed a lot over the last 15 years.

A (partially) updated version of the above survey is available through my webpage:
`http://csseminar.kadm.usu.ru/volkov`

Edmond W. H. Lee and I are working on a new survey (to be submitted this summer).

Advertisement

Semigroups and monoids are the only "classical" algebras for which finite nonfinitely based objects exist: finite groups, associative or Lie rings, lattices are finitely based. Tarski's problem for finite semigroups and monoids still remains open.

There is a survey paper on the FBP for finite semigroups:
M.V. Volkov, The finite basis problem for finite semigroups, *Sci. Math. Jap.*, Vol. 53, 171–199, 2001.

The area has been progressed a lot over the last 15 years.

A (partially) updated version of the above survey is available through my webpage:
`http://csseminar.kadm.usu.ru/volkov`

Edmond W. H. Lee and I are working on a new survey (to be submitted this summer).

Advertisement

Semigroups and monoids are the only "classical" algebras for which finite nonfinitely based objects exist: finite groups, associative or Lie rings, lattices are finitely based. Tarski's problem for finite semigroups and monoids still remains open.

There is a survey paper on the FBP for finite semigroups:
M.V. Volkov, The finite basis problem for finite semigroups, *Sci. Math. Jap.*, Vol. 53, 171–199, 2001.

The area has been progressed a lot over the last 15 years.

A (partially) updated version of the above survey is available through my webpage:
http://csseminar.kadm.usu.ru/volkov

Edmond W. H. Lee and I are working on a new survey (to be submitted this summer).

Advertisement

Semigroups and monoids are the only "classical" algebras for which finite nonfinitely based objects exist: finite groups, associative or Lie rings, lattices are finitely based. Tarski's problem for finite semigroups and monoids still remains open.

There is a survey paper on the FBP for finite semigroups:
M.V. Volkov, The finite basis problem for finite semigroups, *Sci. Math. Jap.*, Vol. 53, 171–199, 2001.

The area has been progressed a lot over the last 15 years.

A (partially) updated version of the above survey is available through my webpage:
http://csseminar.kadm.usu.ru/volkov

Edmond W. H. Lee and I are working on a new survey (to be submitted this summer).

Advertisement

Semigroups and monoids are the only "classical" algebras for which finite nonfinitely based objects exist: finite groups, associative or Lie rings, lattices are finitely based. Tarski's problem for finite semigroups and monoids still remains open.

There is a survey paper on the FBP for finite semigroups:
M.V. Volkov, The finite basis problem for finite semigroups, *Sci. Math. Jap.*, Vol. 53, 171–199, 2001.

The area has been progressed a lot over the last 15 years.

A (partially) updated version of the above survey is available through my webpage:
`http://csseminar.kadm.usu.ru/volkov`

Edmond W. H. Lee and I are working on a new survey (to be submitted this summer).

Advertisement

Semigroups and monoids are the only "classical" algebras for which finite nonfinitely based objects exist: finite groups, associative or Lie rings, lattices are finitely based. Tarski's problem for finite semigroups and monoids still remains open.

There is a survey paper on the FBP for finite semigroups:
M.V. Volkov, The finite basis problem for finite semigroups, *Sci. Math. Jap.*, Vol. 53, 171–199, 2001.

The area has been progressed a lot over the last 15 years.

A (partially) updated version of the above survey is available through my webpage:
`http://csseminar.kadm.usu.ru/volkov`

Edmond W. H. Lee and I are working on a new survey (to be submitted this summer).

FBP for Infinite Semigroups

The FBP for infinite semigroups is less studied.

Reason: "usual" infinite semigroups (transformations, relations, matrices etc) are too big (contain a copy of the non-monogenic free semigroup).

Therefore they satisfy only trivial identities and so they are finitely based in a "void" way.

But if an infinite semigroup does satisfy a non-trivial identity, its FBP constitutes a challenge since no "finite" methods apply.

Aim: to design a new method that would cover interesting classes of infinite semigroups.

FBP for Infinite Semigroups

The FBP for infinite semigroups is less studied.

Reason: "usual" infinite semigroups (transformations, relations, matrices etc) are too big (contain a copy of the non-monogenic free semigroup).

Therefore they satisfy only trivial identities and so they are finitely based in a "void" way.

But if an infinite semigroup does satisfy a non-trivial identity, its FBP constitutes a challenge since no "finite" methods apply.

Aim: to design a new method that would cover interesting classes of infinite semigroups.

FBP for Infinite Semigroups

The FBP for infinite semigroups is less studied.

Reason: "usual" infinite semigroups (transformations, relations, matrices etc) are too big (contain a copy of the non-monogenic free semigroup).

Therefore they satisfy only trivial identities and so they are finitely based in a "void" way.

But if an infinite semigroup does satisfy a non-trivial identity, its FBP constitutes a challenge since no "finite" methods apply.

Aim: to design a new method that would cover interesting classes of infinite semigroups.

FBP for Infinite Semigroups

The FBP for infinite semigroups is less studied.

Reason: "usual" infinite semigroups (transformations, relations, matrices etc) are too big (contain a copy of the non-monogenic free semigroup).

Therefore they satisfy only trivial identities and so they are finitely based in a "void" way.

But if an infinite semigroup does satisfy a non-trivial identity, its FBP constitutes a challenge since no "finite" methods apply.

Aim: to design a new method that would cover interesting classes of infinite semigroups.

FBP for Infinite Semigroups

The FBP for infinite semigroups is less studied.

Reason: “usual” infinite semigroups (transformations, relations, matrices etc) are too big (contain a copy of the non-monogenic free semigroup).

Therefore they satisfy only trivial identities and so they are finitely based in a “void” way.

But if an infinite semigroup does satisfy a non-trivial identity, its FBP constitutes a challenge since no “finite” methods apply.

Aim: to design a new method that would cover interesting classes of infinite semigroups.

Temperley–Lieb Algebras

Neville Temperley and Elliott Lieb ("Relations between the percolation and colouring problem and other graph-theoretical problems associated with regular planar lattices: Some exact results for the percolation problem", *Proc. Roy. Soc. London* Ser. A 322, 251–280, 1971) motivated by some problems in statistical mechanics have introduced what is now called Temperley–Lieb algebras. These are associative linear algebras with 1 over a commutative ring R. Given n and $\delta \in R$, the algebra $TL_n(\delta)$ is generated by $n-1$ generators $h_1, \dots, h_{n-1}$ subject to the relations

$$\begin{aligned} h_i h_j &= h_j h_i \quad \text{if } |i-j| \geq 2, \\ h_i h_j h_i &= h_i \quad \text{if } |i-j| = 1, \\ h_i h_i &= \delta h_i. \end{aligned}$$

Temperley–Lieb Algebras

Neville Temperley and Elliott Lieb ("Relations between the percolation and colouring problem and other graph-theoretical problems associated with regular planar lattices: Some exact results for the percolation problem", *Proc. Roy. Soc. London* Ser. A 322, 251–280, 1971) motivated by some problems in statistical mechanics have introduced what is now called Temperley–Lieb algebras. These are associative linear algebras with 1 over a commutative ring R. Given n and $\delta \in R$, the algebra $TL_n(\delta)$ is generated by $n-1$ generators $h_1, \ldots, h_{n-1}$ subject to the relations

$$\begin{aligned} h_i h_j &= h_j h_i \quad \text{if } |i-j| \geq 2, \\ h_i h_j h_i &= h_i \quad \text{if } |i-j| = 1, \\ h_i h_i &= \delta h_i. \end{aligned}$$

Kauffman Monoids

The relations of $TL_n(\delta)$ do not involve addition. This suggests introducing a monoid whose monoid algebra over R could be identified with $TL_n(\delta)$. A tiny obstacle is the scalar δ in $h_ih_i = \delta h_i$. It can be bypassed by adding a new generator c that imitates δ. This way one gets to the monoid K_n with n generators $c, h_1, \dots, h_{n-1}$ subject to the relations

$$h_ih_j = h_jh_i \quad \text{if } |i-j| \geq 2,$$
$$h_ih_jh_i = h_i \quad \text{if } |i-j| = 1,$$
$$h_ih_i = ch_i = h_ic.$$

The monoids K_n are called the Kauffman monoids. Lois Kauffman ("An invariant of regular isotopy", *Trans. Amer. Math. Soc.* 318, 417–471, 1990) independently invented these monoids as geometrical objects. We present Kauffman's approach in a slightly more general setting.

Kauffman Monoids

The relations of $TL_n(\delta)$ do not involve addition. This suggests introducing a monoid whose monoid algebra over R could be identified with $TL_n(\delta)$. A tiny obstacle is the scalar δ in $h_ih_i = \delta h_i$. It can be bypassed by adding a new generator c that imitates δ. This way one gets to the monoid K_n with n generators $c, h_1, \dots, h_{n-1}$ subject to the relations

$$\begin{aligned} h_ih_j &= h_jh_i \quad \text{if } |i-j| \geq 2, \\ h_ih_jh_i &= h_i \quad \text{if } |i-j| = 1, \\ h_ih_i &= ch_i = h_ic. \end{aligned}$$

The monoids K_n are called the Kauffman monoids. Lois Kauffman ("An invariant of regular isotopy", *Trans. Amer. Math. Soc.* 318, 417–471, 1990) independently invented these monoids as geometrical objects. We present Kauffman's approach in a slightly more general setting.

Kauffman Monoids

The relations of $TL_n(\delta)$ do not involve addition. This suggests introducing a monoid whose monoid algebra over R could be identified with $TL_n(\delta)$. A tiny obstacle is the scalar δ in $h_i h_i = \delta h_i$.
It can be bypassed by adding a new generator c that imitates δ.
This way one gets to the monoid K_n with n generators $c, h_1, \dots, h_{n-1}$ subject to the relations

$$h_i h_j = h_j h_i \quad \text{if } |i-j| \geq 2,$$
$$h_i h_j h_i = h_i \quad \text{if } |i-j| = 1,$$
$$h_i h_i = c h_i = h_i c.$$

The monoids K_n are called the Kauffman monoids. Lois Kauffman ("An invariant of regular isotopy", *Trans. Amer. Math. Soc.* 318, 417–471, 1990) independently invented these monoids as geometrical objects. We present Kauffman's approach in a slightly more general setting.

Kauffman Monoids

The relations of $TL_n(\delta)$ do not involve addition. This suggests introducing a monoid whose monoid algebra over R could be identified with $TL_n(\delta)$. A tiny obstacle is the scalar δ in $h_i h_i = \delta h_i$. It can be bypassed by adding a new generator c that imitates δ.

This way one gets to the monoid K_n with n generators $c, h_1, \dots, h_{n-1}$ subject to the relations

$$\begin{aligned} h_i h_j &= h_j h_i \quad \text{if } |i-j| \geq 2, \\ h_i h_j h_i &= h_i \quad \text{if } |i-j| = 1, \\ h_i h_i &= c h_i = h_i c. \end{aligned}$$

The monoids K_n are called the Kauffman monoids. Lois Kauffman ("An invariant of regular isotopy", *Trans. Amer. Math. Soc.* 318, 417–471, 1990) independently invented these monoids as geometrical objects. We present Kauffman's approach in a slightly more general setting.

Kauffman Monoids

The relations of $TL_n(\delta)$ do not involve addition. This suggests introducing a monoid whose monoid algebra over R could be identified with $TL_n(\delta)$. A tiny obstacle is the scalar δ in $h_ih_i = \delta h_i$. It can be bypassed by adding a new generator c that imitates δ. This way one gets to the monoid K_n with n generators $c, h_1, \ldots, h_{n-1}$ subject to the relations

$$\begin{aligned} h_ih_j &= h_jh_i \quad \text{if } |i-j| \geq 2, \\ h_ih_jh_i &= h_i \quad \text{if } |i-j| = 1, \\ h_ih_i &= ch_i = h_ic. \end{aligned}$$

The monoids K_n are called the Kauffman monoids. Lois Kauffman ("An invariant of regular isotopy", *Trans. Amer. Math. Soc.* 318, 417–471, 1990) independently invented these monoids as geometrical objects. We present Kauffman's approach in a slightly more general setting.

Kauffman Monoids

The relations of $TL_n(\delta)$ do not involve addition. This suggests introducing a monoid whose monoid algebra over R could be identified with $TL_n(\delta)$. A tiny obstacle is the scalar δ in $h_i h_i = \delta h_i$. It can be bypassed by adding a new generator c that imitates δ. This way one gets to the monoid K_n with n generators $c, h_1, \dots, h_{n-1}$ subject to the relations

$$h_i h_j = h_j h_i \quad \text{if } |i-j| \geq 2,$$
$$h_i h_j h_i = h_i \quad \text{if } |i-j| = 1,$$
$$h_i h_i = c h_i = h_i c.$$

The monoids K_n are called the Kauffman monoids. Lois Kauffman ("An invariant of regular isotopy", *Trans. Amer. Math. Soc.* 318, 417–471, 1990) independently invented these monoids as geometrical objects. We present Kauffman's approach in a slightly more general setting.

Kauffman Monoids

The relations of $TL_n(\delta)$ do not involve addition. This suggests introducing a monoid whose monoid algebra over R could be identified with $TL_n(\delta)$. A tiny obstacle is the scalar δ in $h_ih_i = \delta h_i$. It can be bypassed by adding a new generator c that imitates δ. This way one gets to the monoid K_n with n generators $c, h_1, \dots, h_{n-1}$ subject to the relations

$$\begin{aligned} h_ih_j &= h_jh_i \quad \text{if } |i-j| \geq 2, \\ h_ih_jh_i &= h_i \quad \text{if } |i-j| = 1, \\ h_ih_i &= ch_i = h_ic. \end{aligned}$$

The monoids K_n are called the Kauffman monoids. Lois Kauffman ("An invariant of regular isotopy", *Trans. Amer. Math. Soc.* 318, 417–471, 1990) independently invented these monoids as geometrical objects. We present Kauffman's approach in a slightly more general setting.

Kauffman Monoids

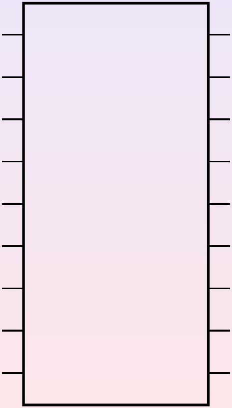
The relations of $TL_n(\delta)$ do not involve addition. This suggests introducing a monoid whose monoid algebra over R could be identified with $TL_n(\delta)$. A tiny obstacle is the scalar δ in $h_i h_i = \delta h_i$. It can be bypassed by adding a new generator c that imitates δ. This way one gets to the monoid K_n with n generators $c, h_1, \ldots, h_{n-1}$ subject to the relations

$$
\begin{aligned}
h_i h_j &= h_j h_i \quad \text{if } |i-j| \geq 2, \\
h_i h_j h_i &= h_i \quad \text{if } |i-j| = 1, \\
h_i h_i &= c h_i = h_i c.
\end{aligned}
$$

The monoids K_n are called the Kauffman monoids. Lois Kauffman ("An invariant of regular isotopy", *Trans. Amer. Math. Soc.* 318, 417–471, 1990) independently invented these monoids as geometrical objects. We present Kauffman's approach in a slightly more general setting.

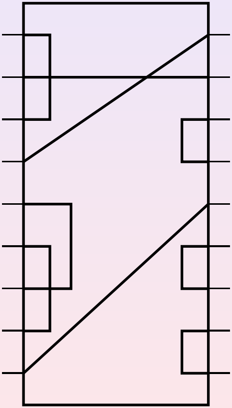
Wire Monoids

Fix n and consider "chips" with $2n$ pins, n on each side. Pins are connected by n wires. To multiply two chips, we connect the right hand side pins of the first with the left hand side pins of the second.



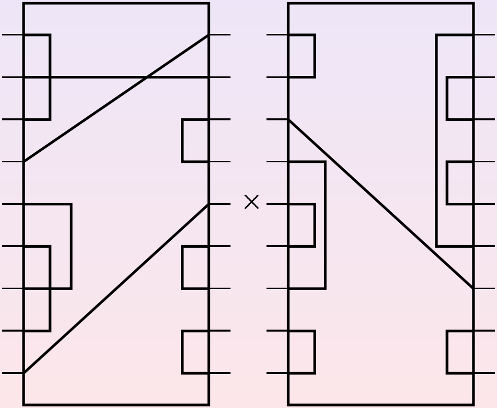
Wire Monoids

Fix n and consider "chips" with $2n$ pins, n on each side. Pins are connected by n wires. To multiply two chips, we connect the right hand side pins of the first with the left hand side pins of the second.



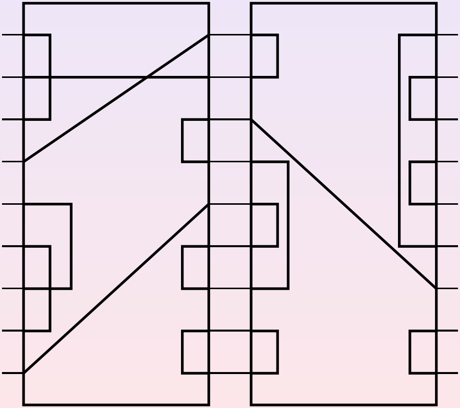
Wire Monoids

Fix n and consider "chips" with $2n$ pins, n on each side. Pins are connected by n wires. To multiply two chips, we connect the right hand side pins of the first with the left hand side pins of the second.



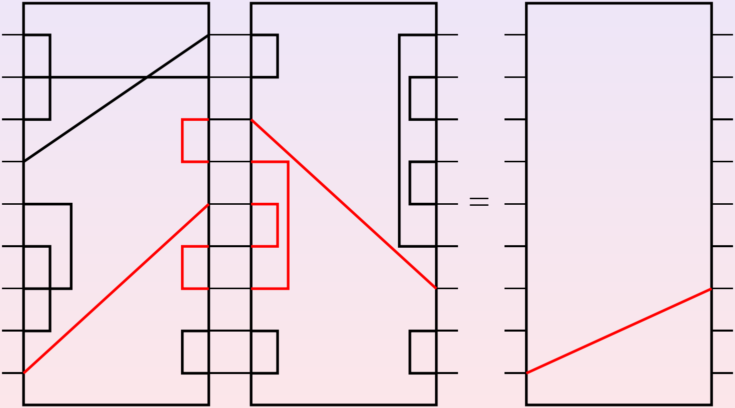
Wire Monoids

Fix n and consider "chips" with $2n$ pins, n on each side. Pins are connected by n wires. To multiply two chips, we connect the right hand side pins of the first with the left hand side pins of the second.



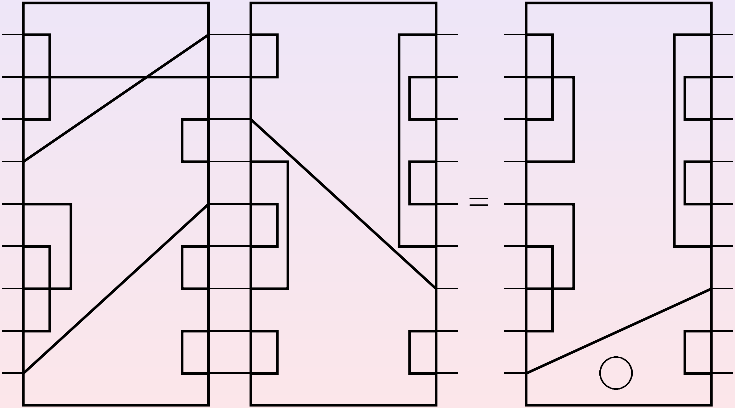
Wire Monoids

Fix n and consider "chips" with $2n$ pins, n on each side. Pins are connected by n wires. To multiply two chips, we connect the right hand side pins of the first with the left hand side pins of the second.



Wire Monoids

Fix n and consider "chips" with $2n$ pins, n on each side. Pins are connected by n wires. To multiply two chips, we connect the right hand side pins of the first with the left hand side pins of the second.

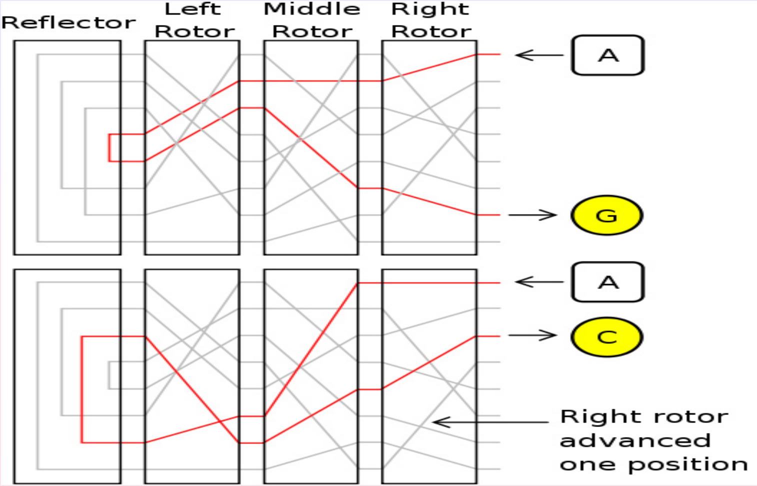


Enigma as a Wire Monoid

The multiplication rule resembles the Enigma machine of WWII:

Enigma as a Wire Monoid

The multiplication rule resembles the Enigma machine of WWII:



Types of Wire Monoids

There are two issues on which the outcome of the above definition depends: 1) do we care of crossing wires? 2) do we care of circles?

	Ignore circles	Count circles
Crossings OK	Brauer monoids	Wire monoids
No crossings	Jones monoids	Kauffman monoids

Types of Wire Monoids

There are two issues on which the outcome of the above definition depends: 1) do we care of crossing wires? 2) do we care of circles?

	Ignore circles	Count circles
Crossings OK	Brauer monoids	Wire monoids
No crossings	Jones monoids	Kauffman monoids

Types of Wire Monoids

There are two issues on which the outcome of the above definition depends: 1) do we care of crossing wires? 2) do we care of circles?

	Ignore circles	Count circles
Crossings OK	Brauer monoids	Wire monoids
No crossings	Jones monoids	Kauffman monoids

Types of Wire Monoids

There are two issues on which the outcome of the above definition depends: 1) do we care of crossing wires? 2) do we care of circles?

	Ignore circles	Count circles
Crossings OK	Brauer monoids	Wire monoids
No crossings	Jones monoids	Kauffman monoids

Richard Brauer's monoids arose in his paper 'On algebras which are connected with the semisimple continuous groups', *Ann. Math.* 38, 857–872, 1937 as vector space bases of certain associative algebras relevant in representation theory.

Types of Wire Monoids

There are two issues on which the outcome of the above definition depends: 1) do we care of crossing wires? 2) do we care of circles?

	Ignore circles	Count circles
Crossings OK	Brauer monoids	Wire monoids
No crossings	Jones monoids	Kauffman monoids

Jones monoids are named after Vaughan Jones, the famous knot theorist. We denote by J_n the Jones monoid of chips with n pins.

Types of Wire Monoids

There are two issues on which the outcome of the above definition depends: 1) do we care of crossing wires? 2) do we care of circles?

	Ignore circles	Count circles
Crossings OK	Brauer monoids	Wire monoids
No crossings	Jones monoids	Kauffman monoids

Types of Wire Monoids

There are two issues on which the outcome of the above definition depends: 1) do we care of crossing wires? 2) do we care of circles?

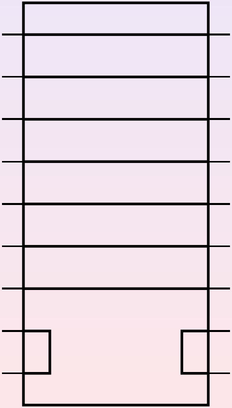
	Ignore circles	Count circles
Crossings OK	Brauer monoids	Wire monoids
No crossings	Jones monoids	Kauffman monoids

Kauffman Monoids as Wire Monoids

Thus the Kauffman monoid K_n consists of $2n$-pin chips with non-crossing wires that may contain circles. Only the number of circles matters, not their location. The monoid K_n is generated by the hooks $h_1, \dots, h_{n-1}$ and the circle c.

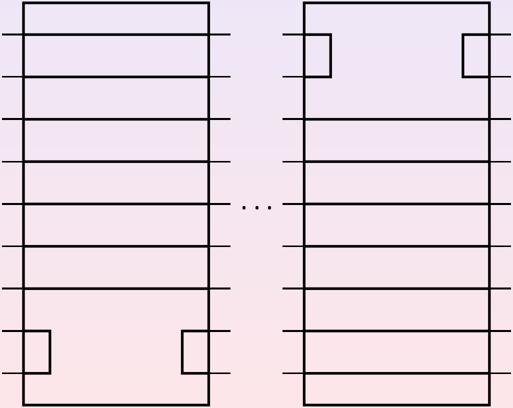
Kauffman Monoids as Wire Monoids

Thus the Kauffman monoid K_n consists of $2n$-pin chips with non-crossing wires that may contain circles. Only the number of circles matters, not their location. The monoid K_n is generated by the hooks $h_1, \ldots, h_{n-1}$ and the circle c.



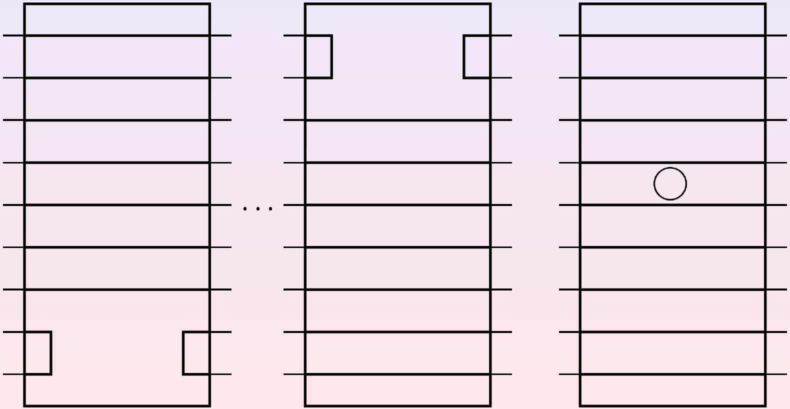
Kauffman Monoids as Wire Monoids

Thus the Kauffman monoid K_n consists of $2n$-pin chips with non-crossing wires that may contain circles. Only the number of circles matters, not their location. The monoid K_n is generated by the hooks $h_1, \ldots, h_{n-1}$ and the circle c.



Kauffman Monoids as Wire Monoids

Thus the Kauffman monoid K_n consists of $2n$-pin chips with non-crossing wires that may contain circles. Only the number of circles matters, not their location. The monoid K_n is generated by the hooks $h_1, \dots, h_{n-1}$ and the circle c.



Kauffman Monoids as Wire Monoids contnd

Recall the relations we used to define K_n:

$$
\begin{aligned}
h_i h_j &= h_j h_i \quad \text{if } |i-j| \geq 2,\\
h_i h_j h_i &= h_i \quad \text{if } |i-j| = 1,\\
h_i h_i &= c h_i,\\
c h_i &= h_i c.
\end{aligned}
$$

These relations are satisfied when h_i and c are interpreted as the hooks and the circle. For the last relation it is clear—the circle does not react with the hooks, for the others it is shown in the next slides.

Kauffman Monoids as Wire Monoids contnd

Recall the relations we used to define K_n:

$$
\begin{aligned}
h_i h_j &= h_j h_i \quad \text{if } |i-j| \geq 2,\\
h_i h_j h_i &= h_i \quad \text{if } |i-j| = 1,\\
h_i h_i &= c h_i,\\
c h_i &= h_i c.
\end{aligned}
$$

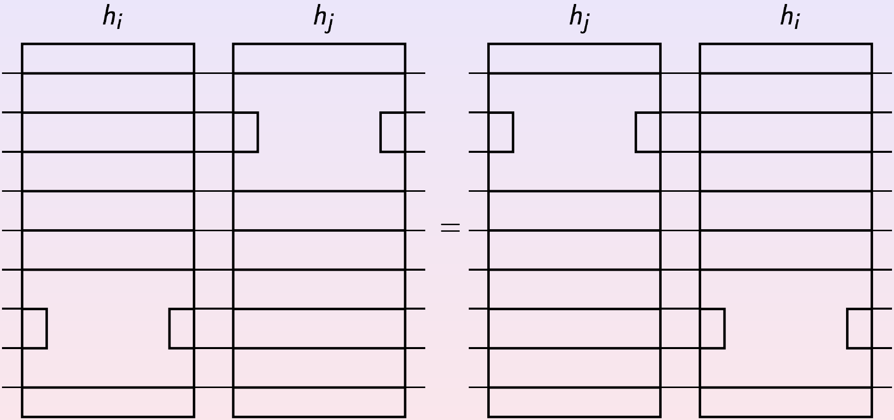
These relations are satisfied when h_i and c are interpreted as the hooks and the circle. For the last relation it is clear—the circle does not react with the hooks, for the others it is shown in the next slides.

Recall the relations we used to define K_n:

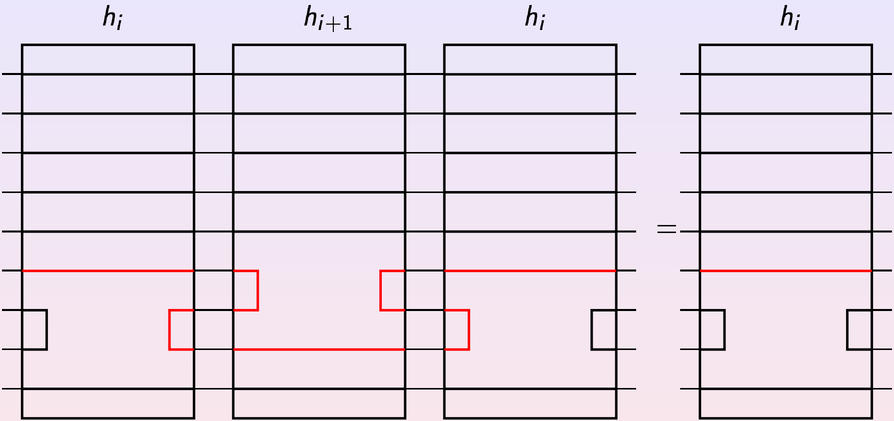
$$h_i h_j = h_j h_i \quad \text{if } |i - j| \geq 2,$$
$$h_i h_j h_i = h_i \quad \text{if } |i - j| = 1,$$
$$h_i h_i = c h_i,$$
$$c h_i = h_i c.$$

These relations are satisfied when h_i and c are interpreted as the hooks and the circle. For the last relation it is clear—the circle does not react with the hooks, for the others it is shown in the next slides.

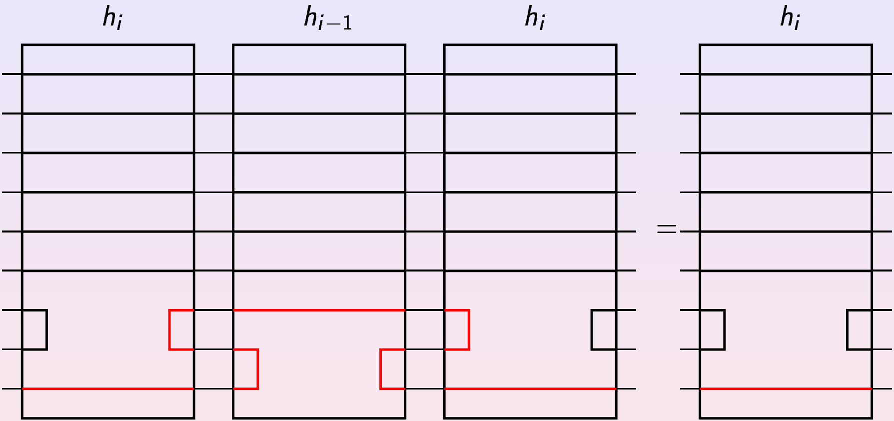
$h_i h_j = h_j h_i$ if $|i - j| \geq 2$



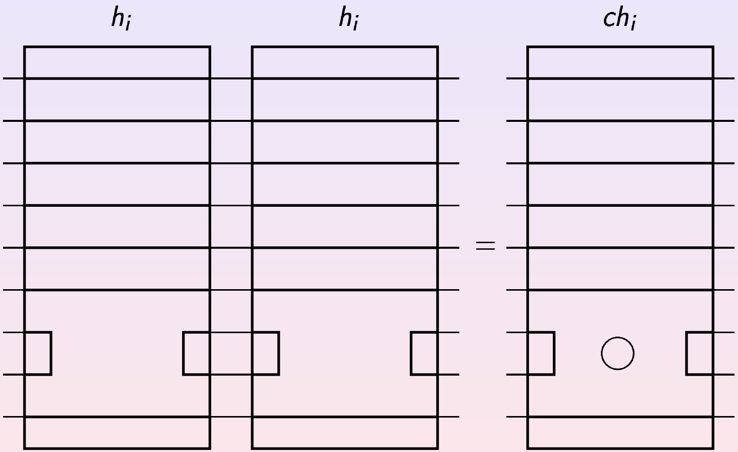
$h_i h_j h_i = h_i$ if $|i - j| = 1$



$h_i h_j h_i = h_i$ if $|i - j| = 1$



$h_i h_i = ch_i$



Kauffman Monoids as Wire Monoids contnd

Thus, the “planar” wire monoid generated by the hooks and the circle satisfies the relation of K_n and is therefore a homomorphic image of K_n. In fact, this wire monoid is isomorphic to K_n (requires some work). This connection was realized by Jones who didn't bother himself with a formal proof. Such a proof has first been published by Mirjana Borisavljević, Kosta Došen and Zoran Petrić (Kauffman monoids, *J. Knot Theory Ramifications* 11, 127–143, 2002).

Similarly, one can show that the Jones monoid J_n is generated by the hooks $h_1, \ldots, h_{n-1}$ subject the relations

$$h_i h_j = h_j h_i \quad \text{if } |i-j| \geq 2,$$
$$h_i h_j h_i = h_i \quad \text{if } |i-j| = 1,$$
$$h_i h_i = h_i.$$

Thus, J_n spans the Temperley-Lieb algebra $TL_n(1)$.

Kauffman Monoids as Wire Monoids contnd

Thus, the "planar" wire monoid generated by the hooks and the circle satisfies the relation of K_n and is therefore a homomorphic image of K_n. In fact, this wire monoid is isomorphic to K_n (requires some work). This connection was realized by Jones who didn't bother himself with a formal proof. Such a proof has first been published by Mirjana Borisavljević, Kosta Došen and Zoran Petrić (Kauffman monoids, *J. Knot Theory Ramifications* 11, 127–143, 2002).

Similarly, one can show that the Jones monoid J_n is generated by the hooks $h_1, \ldots, h_{n-1}$ subject the relations

$$h_i h_j = h_j h_i \quad \text{if } |i-j| \geq 2,$$
$$h_i h_j h_i = h_i \quad \text{if } |i-j| = 1,$$
$$h_i h_i = h_i.$$

Thus, J_n spans the Temperley-Lieb algebra $TL_n(1)$.

Kauffman Monoids as Wire Monoids contnd

Thus, the "planar" wire monoid generated by the hooks and the circle satisfies the relation of K_n and is therefore a homomorphic image of K_n. In fact, this wire monoid is isomorphic to K_n (requires some work). This connection was realized by Jones who didn't bother himself with a formal proof. Such a proof has first been published by Mirjana Borisavljević, Kosta Došen and Zoran Petrić (Kauffman monoids, *J. Knot Theory Ramifications* 11, 127–143, 2002).

Similarly, one can show that the Jones monoid J_n is generated by the hooks $h_1, \ldots, h_{n-1}$ subject the relations

$$h_i h_j = h_j h_i \quad \text{if } |i - j| \geq 2,$$
$$h_i h_j h_i = h_i \quad \text{if } |i - j| = 1,$$
$$h_i h_i = h_i.$$

Thus, J_n spans the Temperley-Lieb algebra $TL_n(1)$.

Kauffman Monoids as Wire Monoids contnd

Thus, the "planar" wire monoid generated by the hooks and the circle satisfies the relation of K_n and is therefore a homomorphic image of K_n. In fact, this wire monoid is isomorphic to K_n (requires some work). This connection was realized by Jones who didn't bother himself with a formal proof. Such a proof has first been published by Mirjana Borisavljević, Kosta Došen and Zoran Petrić (Kauffman monoids, *J. Knot Theory Ramifications* 11, 127–143, 2002).

Similarly, one can show that the Jones monoid J_n is generated by the hooks $h_1, \dots, h_{n-1}$ subject the relations

$$
\begin{aligned}
h_i h_j &= h_j h_i \quad \text{if } |i-j| \geq 2,\\
h_i h_j h_i &= h_i \quad \text{if } |i-j| = 1,\\
h_i h_i &= h_i.
\end{aligned}
$$

Thus, J_n spans the Temperley-Lieb algebra $TL_n(1)$.

Kauffman Monoids as Wire Monoids contnd

Thus, the "planar" wire monoid generated by the hooks and the circle satisfies the relation of K_n and is therefore a homomorphic image of K_n. In fact, this wire monoid is isomorphic to K_n (requires some work). This connection was realized by Jones who didn't bother himself with a formal proof. Such a proof has first been published by Mirjana Borisavljević, Kosta Došen and Zoran Petrić (Kauffman monoids, *J. Knot Theory Ramifications* 11, 127–143, 2002).

Similarly, one can show that the Jones monoid J_n is generated by the hooks $h_1, \dots, h_{n-1}$ subject the relations

$$\begin{aligned} h_i h_j &= h_j h_i \quad \text{if } |i-j| \geq 2, \\ h_i h_j h_i &= h_i \quad \text{if } |i-j| = 1, \\ h_i h_i &= h_i. \end{aligned}$$

Thus, J_n spans the Temperley-Lieb algebra $TL_n(1)$.

Kauffman Monoids as Wire Monoids contnd

Thus, the "planar" wire monoid generated by the hooks and the circle satisfies the relation of K_n and is therefore a homomorphic image of K_n. In fact, this wire monoid is isomorphic to K_n (requires some work). This connection was realized by Jones who didn't bother himself with a formal proof. Such a proof has first been published by Mirjana Borisavljević, Kosta Došen and Zoran Petrić (Kauffman monoids, *J. Knot Theory Ramifications* 11, 127–143, 2002).
Similarly, one can show that the Jones monoid J_n is generated by the hooks $h_1, \dots, h_{n-1}$ subject the relations

$$\begin{aligned} h_i h_j &= h_j h_i \quad \text{if } |i-j| \geq 2, \\ h_i h_j h_i &= h_i \quad \text{if } |i-j| = 1, \\ h_i h_i &= h_i. \end{aligned}$$

Thus, J_n spans the Temperley-Lieb algebra $TL_n(1)$.

Involutions

The full wire monoid admits two natural unary operations: reflection (each chip reflects chip along its vertical symmetry axis) and rotation (each chip rotates by the angle of 180 degrees). Both reflection and rotation are easily seen to be involutions, i.e.,

$$(xy)^* = y^*x^* \text{ and } x^{**} = x.$$

Clearly, the Kauffman monoid K_n is closed under each of the two involutions.

Involutions

The full wire monoid admits two natural unary operations: reflection (each chip reflects chip along its vertical symmetry axis) and rotation (each chip rotates by the angle of 180 degrees). Both reflection and rotation are easily seen to be involutions, i.e.,

$$(xy)^* = y^*x^* \text{ and } x^{**} = x.$$

Clearly, the Kauffman monoid K_n is closed under each of the two involutions.

Involutions

The full wire monoid admits two natural unary operations: reflection (each chip reflects chip along its vertical symmetry axis) and rotation (each chip rotates by the angle of 180 degrees).

Both reflection and rotation are easily seen to be involutions, i.e.,

$$(xy)^* = y^*x^* \text{ and } x^{**} = x.$$

Clearly, the Kauffman monoid K_n is closed under each of the two involutions.

Involutions

The full wire monoid admits two natural unary operations: reflection (each chip reflects chip along its vertical symmetry axis) and rotation (each chip rotates by the angle of 180 degrees). Both reflection and rotation are easily seen to be involutions, i.e.,

$$(xy)^* = y^*x^* \text{ and } x^{**} = x.$$

Clearly, the Kauffman monoid K_n is closed under each of the two involutions.

Involutions

The full wire monoid admits two natural unary operations: reflection (each chip reflects chip along its vertical symmetry axis) and rotation (each chip rotates by the angle of 180 degrees). Both reflection and rotation are easily seen to be involutions, i.e.,

$$(xy)^* = y^*x^* \text{ and } x^{**} = x.$$

Clearly, the Kauffman monoid K_n is closed under each of the two involutions.

Theorem

For each $n \geq 3$, the Kauffman monoid K_n is nonfinitely based both as a semigroup and as an involution semigroup (with either of the two natural involutions).

The Kauffman monoid K_2 is commutative and hence is finitely based; also as an involution semigroup. Hence we have a complete solution to the FBP for the Kauffman monoids in both plain and unary settings.

The fact that the monoid K_n with $n \geq 4$ is nonfinitely based was announced in my lecture at the 3rd Novi Sad Algebraic Conference in 2009. The case $n = 3$ was left open and so was the question about the FBP for K_n as an involution monoid with respect to reflection. Now these two questions have been settled + we have solved also the FBP for K_n as an involution monoid with respect to rotation.

FBP in Kauffman Monoids

Theorem

For each $n \geq 3$, the Kauffman monoid K_n is nonfinitely based both as a semigroup and as an involution semigroup (with either of the two natural involutions).

The Kauffman monoid K_2 is commutative and hence is finitely based; also as an involution semigroup. Hence we have a complete solution to the FBP for the Kauffman monoids in both plain and unary settings.

The fact that the monoid K_n with $n \geq 4$ is nonfinitely based was announced in my lecture at the 3rd Novi Sad Algebraic Conference in 2009. The case $n = 3$ was left open and so was the question about the FBP for K_n as an involution monoid with respect to reflection. Now these two questions have been settled + we have solved also the FBP for K_n as an involution monoid with respect to rotation.

FBP in Kauffman Monoids

Theorem

For each $n \geq 3$, the Kauffman monoid K_n is nonfinitely based both as a semigroup and as an involution semigroup (with either of the two natural involutions).

The Kauffman monoid K_2 is commutative and hence is finitely based; also as an involution semigroup. Hence we have a complete solution to the FBP for the Kauffman monoids in both plain and unary settings.

The fact that the monoid K_n with $n \geq 4$ is nonfinitely based was announced in my lecture at the 3rd Novi Sad Algebraic Conference in 2009. The case $n = 3$ was left open and so was the question about the FBP for K_n as an involution monoid with respect to reflection. Now these two questions have been settled + we have solved also the FBP for K_n as an involution monoid with respect to rotation.

Theorem

For each $n \geq 3$, the Kauffman monoid K_n is nonfinitely based both as a semigroup and as an involution semigroup (with either of the two natural involutions).

The Kauffman monoid K_2 is commutative and hence is finitely based; also as an involution semigroup. Hence we have a complete solution to the FBP for the Kauffman monoids in both plain and unary settings.

The fact that the monoid K_n with $n \geq 4$ is nonfinitely based was announced in my lecture at the 3rd Novi Sad Algebraic Conference in 2009. The case $n = 3$ was left open and so was the question about the FBP for K_n as an involution monoid with respect to reflection. Now these two questions have been settled + we have solved also the FBP for K_n as an involution monoid with respect to rotation.

FBP in Kauffman Monoids

Theorem

For each $n \geq 3$, the Kauffman monoid K_n is nonfinitely based both as a semigroup and as an involution semigroup (with either of the two natural involutions).

The Kauffman monoid K_2 is commutative and hence is finitely based; also as an involution semigroup. Hence we have a complete solution to the FBP for the Kauffman monoids in both plain and unary settings.

The fact that the monoid K_n with $n \geq 4$ is nonfinitely based was announced in my lecture at the 3rd Novi Sad Algebraic Conference in 2009. The case $n = 3$ was left open and so was the question about the FBP for K_n as an involution monoid with respect to reflection. Now these two questions have been settled + we have solved also the FBP for K_n as an involution monoid with respect to rotation.

FBP in Kauffman Monoids

Theorem

For each $n \geq 3$, the Kauffman monoid K_n is nonfinitely based both as a semigroup and as an involution semigroup (with either of the two natural involutions).

The Kauffman monoid K_2 is commutative and hence is finitely based; also as an involution semigroup. Hence we have a complete solution to the FBP for the Kauffman monoids in both plain and unary settings.

The fact that the monoid K_n with $n \geq 4$ is nonfinitely based was announced in my lecture at the 3rd Novi Sad Algebraic Conference in 2009. The case $n = 3$ was left open and so was the question about the FBP for K_n as an involution monoid with respect to reflection. Now these two questions have been settled + we have solved also the FBP for K_n as an involution monoid with respect to rotation.

Further Applications

In fact, we have found a new sufficient condition under which a semigroup (or an involution semigroup) is nonfinitely based. When specialized to finite semigroups, it gives a well-known condition for a being inherently nonfinitely based. But there are many further applications to various classes of infinite semigroups.

Further Applications

In fact, we have found a new sufficient condition under which a semigroup (or an involution semigroup) is nonfinitely based. When specialized to finite semigroups, it gives a well-known condition for a being inherently nonfinitely based. But there are many further applications to various classes of infinite semigroups.

Further Applications

In fact, we have found a new sufficient condition under which a semigroup (or an involution semigroup) is nonfinitely based. When specialized to finite semigroups, it gives a well-known condition for a being inherently nonfinitely based. But there are many further applications to various classes of infinite semigroups.

Further Applications

In fact, we have found a new sufficient condition under which a semigroup (or an involution semigroup) is nonfinitely based. When specialized to finite semigroups, it gives a well-known condition for a being inherently nonfinitely based. But there are many further applications to various classes of infinite semigroups.

Theorem

For each $n \geq 3$, the full wire monoid (the monoid of all $2n$-pin chips) is nonfinitely based both as a semigroup and as an involution semigroup (with either of the two natural involutions).

Further Applications

In fact, we have found a new sufficient condition under which a semigroup (or an involution semigroup) is nonfinitely based. When specialized to finite semigroups, it gives a well-known condition for a being inherently nonfinitely based. But there are many further applications to various classes of infinite semigroups.

Theorem

For each $n \geq 3$, the full wire monoid (the monoid of all $2n$-pin chips) is nonfinitely based both as a semigroup and as an involution semigroup (with either of the two natural involutions).

Theorem

The local monoids of the category of 2-cobordisms between 1-dimensional manifolds are nonfinitely based.

Secret Slide

Let S be a semigroup [with involution] such that:

- [the semigroup reduct of] S belongs to the Mal'cev product of a variety whose periodic semigroups are locally finite with a locally finite variety;
- each Zimin word Z_m is an [involution] isoterm relative to S.

Then S is nonfinitely based [as involution semigroup].

The Mal'cev product of varieties $\mathbf{V}$ and $\mathbf{W}$ is the class of all algebras A possessing a congruence θ such that A/θ lies in $\mathbf{W}$ while each θ-class being a subalgebra of A belongs to $\mathbf{V}$.

The Zimin words Z_m are defined as follows: $Z_1 = x_1$, ..., $Z_m = Z_{m-1}x_mZ_{m-1}$.

A term t is an isoterm relative to an algebra A if the only term t' such that the identity $t = t'$ holds in A is t itself.

Let S be a semigroup [with involution] such that:

- [the semigroup reduct of] S belongs to the Mal'cev product of a variety whose periodic semigroups are locally finite with a locally finite variety;
- each Zimin word Z_m is an [involution] isoterm relative to S.

Then S is nonfinitely based [as involution semigroup].

The Mal'cev product of varieties $\mathbf{V}$ and $\mathbf{W}$ is the class of all algebras A possessing a congruence θ such that A/θ lies in $\mathbf{W}$ while each θ-class being a subalgebra of A belongs to $\mathbf{V}$.

The Zimin words Z_m are defined as follows: $Z_1 = x_1, \ldots,$ $Z_m = Z_{m-1}x_mZ_{m-1}$.

A term t is an isoterm relative to an algebra A if the only term t' such that the identity $t = t'$ holds in A is t itself.

Let S be a semigroup [with involution] such that:

- [the semigroup reduct of] S belongs to the Mal'cev product of a variety whose periodic semigroups are locally finite with a locally finite variety;
- each Zimin word Z_m is an [involution] isoterm relative to S.

Then S is nonfinitely based [as involution semigroup].

The Mal'cev product of varieties **V** and **W** is the class of all algebras A possessing a congruence θ such that A/θ lies in **W** while each θ-class being a subalgebra of A belongs to **V**.

The Zimin words Z_m are defined as follows: $Z_1 = x_1, \ldots,$ $Z_m = Z_{m-1}x_mZ_{m-1}$.

A term t is an isoterm relative to an algebra A if the only term t' such that the identity $t = t'$ holds in A is t itself.

Secret Slide

Let S be a semigroup [with involution] such that:

- [the semigroup reduct of] S belongs to the Mal'cev product of a variety whose periodic semigroups are locally finite with a locally finite variety;
- each Zimin word Z_m is an [involution] isoterm relative to S.

Then S is nonfinitely based [as involution semigroup].

The Mal'cev product of varieties $\mathbf{V}$ and $\mathbf{W}$ is the class of all algebras A possessing a congruence θ such that A/θ lies in $\mathbf{W}$ while each θ-class being a subalgebra of A belongs to $\mathbf{V}$.

The Zimin words Z_m are defined as follows: $Z_1 = x_1, \dots,$ $Z_m = Z_{m-1}x_mZ_{m-1}$.

A term t is an isoterm relative to an algebra A if the only term t' such that the identity $t = t'$ holds in A is t itself.

Secret Slide

Let S be a semigroup [with involution] such that:

- [the semigroup reduct of] S belongs to the Mal'cev product of a variety whose periodic semigroups are locally finite with a locally finite variety;
- each Zimin word Z_m is an [involution] isoterm relative to S.

Then S is nonfinitely based [as involution semigroup].

The Mal'cev product of varieties $\mathbf{V}$ and $\mathbf{W}$ is the class of all algebras A possessing a congruence θ such that A/θ lies in $\mathbf{W}$ while each θ-class being a subalgebra of A belongs to $\mathbf{V}$.

The Zimin words Z_m are defined as follows: $Z_1 = x_1, \ldots,$ $Z_m = Z_{m-1}x_mZ_{m-1}$.

A term t is an isoterm relative to an algebra A if the only term t' such that the identity $t = t'$ holds in A is t itself.

Secret Slide

Let S be a semigroup [with involution] such that:

- [the semigroup reduct of] S belongs to the Mal'cev product of a variety whose periodic semigroups are locally finite with a locally finite variety;
- each Zimin word Z_m is an [involution] isoterm relative to S.

Then S is nonfinitely based [as involution semigroup].

The Mal'cev product of varieties $\mathbf{V}$ and $\mathbf{W}$ is the class of all algebras A possessing a congruence θ such that A/θ lies in $\mathbf{W}$ while each θ-class being a subalgebra of A belongs to $\mathbf{V}$.

The Zimin words Z_m are defined as follows: $Z_1 = x_1, \dots,$ $Z_m = Z_{m-1}x_mZ_{m-1}$.

A term t is an isoterm relative to an algebra A if the only term t' such that the identity $t = t'$ holds in A is t itself.