# Open Problems on Synchronizing Automata

Mikhail Volkov

Ural State University, Ekaterinburg, Russia

SATA, Sept 10th, 2008

# Summary

- Classical Problems and Their Restricted Versions
- Algorithmic and Complexity Problems
- Problems Dealing with Synchronizing Digraphs

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# Recap on Synchronization

**Definition**

A DFA $\mathscr{A} = \langle Q, \Sigma, \delta \rangle$ is called synchronizing if there exists a word $w \in \Sigma^*$ whose action resets $\mathscr{A}$, that is, leaves the automaton in one particular state no matter which state in $Q$ it started at: $\delta(q, w) = \delta(q', w)$ for all $q, q' \in Q$.

In symbols, $|\delta(Q, w)| = 1$. Here $\delta(Q, w) = \{\delta(q, w) \mid q \in Q\}$.
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# Classical Problems

- The Černý Conjecture: is it true that, for any synchronizing automaton with $n$ states, there exists a reset word of length $(n-1)^2$?
- The Rank Conjecture: is it true that, for any DFA $\mathscr{A} = \langle Q, \Sigma, \delta \rangle$ with $n$ states and rank $k$, there is a $w \in \Sigma^*$ of length $(n-k)^2$ such that $|\delta(Q, w)| = k$?

The rank of a DFA $\mathscr{A} = \langle Q, \Sigma, \delta \rangle$ is the minimum cardinality of the sets $\delta(Q, w)$ where $w$ runs over $\Sigma^*$. This is the minimum score that can be achieved in the solitaire game on the automaton $\mathscr{A}$. Synchronizing automata are precisely those of rank 1.

- The Černý function's behaviour for various restricted classes of synchronizing automata.

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# Aperiodic Automata

Example: Recently Trahtman has proved that every synchronizing aperiodic automaton with $n$ states admits a reset word of length at most $\frac{n(n-1)}{2}$.

However no precise bound for SAS($n$), the minimum length of reset words for synchronizing aperiodic automata with $n$ states, has been found so far.

(Trahtman) $\frac{n(n-1)}{2} \geq \mathrm{SAS}(n) \geq n + \left\lfloor \frac{n}{2} \right\rfloor - 2$ (Ananichev)

The gap between the upper and the lower bounds is rather drastic. Similar things happen for several other classes of synchronizing automata for whose Černý functions we know an upper bound (Eulerian, weakly monotonic).

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# Letters of Deficiency 2

Let $\mathscr{A} = \langle Q, \Sigma, \delta \rangle$ be a DFA with $|Q| > 3$. The deficiency of a word $v \in \Sigma^*$ w.r.t. $\mathscr{A}$ is the difference $\mathrm{df}_{\mathscr{A}}(v) = |Q| - |\delta(Q, v)|$. If a letter $a \in \Sigma$ has deficiency 2, then exactly one of the two following situations happens:

1. There are three different states $q_1, q_2, q_3 \in Q$ such that

$$\delta(q_1, a) = \delta(q_2, a) = \delta(q_3, a).$$

2. There are four different states $q_1, q_2, q_3, q_4 \in Q$ such that

$$\delta(q_1, a) = \delta(q_2, a) \neq \delta(q_3, a) = \delta(q_4, a).$$

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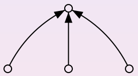
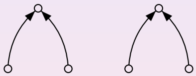
In the first situation we say that $a$ is a dromedary letter, and in the second case we say that $a$ is a bactrian letter.



The action of a dromedary letter



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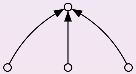

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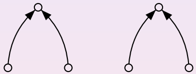

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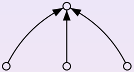

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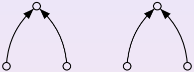

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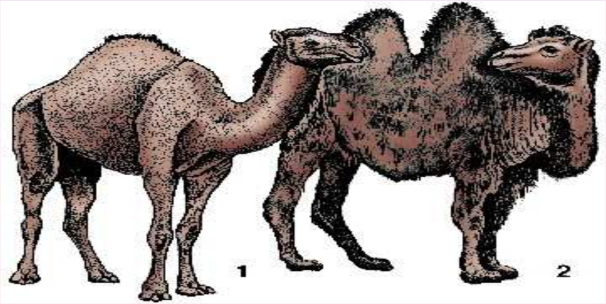

# Letters of Deficiency 2

For each $n > 4$, there is a synchronizing automaton with $n$ states and 3 input letters one of which is dromedary whose shortest reset word is of length $(n-2)^2+1$.

For each odd $n > 3$, there is a synchronizing automaton with $n$ states and 2 input letters one of which is bactrian whose shortest reset word is of length $(n-1)(n-2)$.

Do these lower bounds represent the worst possible case? In other words, is the Černý function for the class of synchronizing automata with a dromedary/bactrian letter equal to $(n-2)^2+1$/respectively $(n-1)(n-2)$?

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# Letters of Deficiency $k$

For each $n > k + 1$ not divisible by $k$, there is a synchronizing automaton with $n$ states and 2 input letters one of which has deficiency $k$ whose shortest reset word is of length $(n-1)(n-k)$.

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Observe that again the original Černý conjecture correspond to the case $k = 1$.

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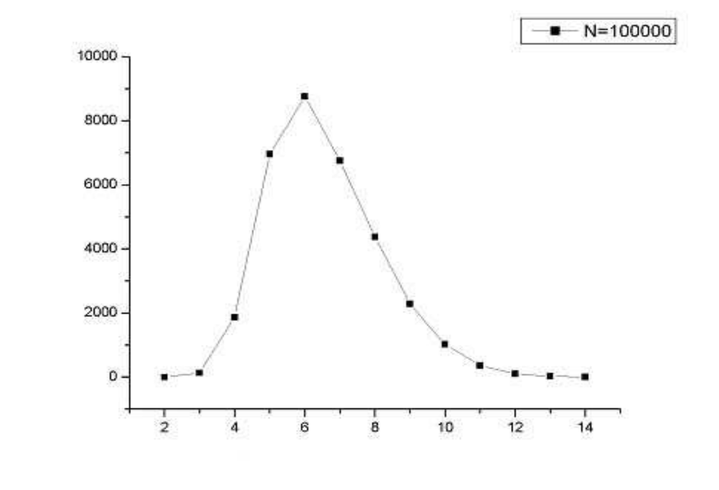
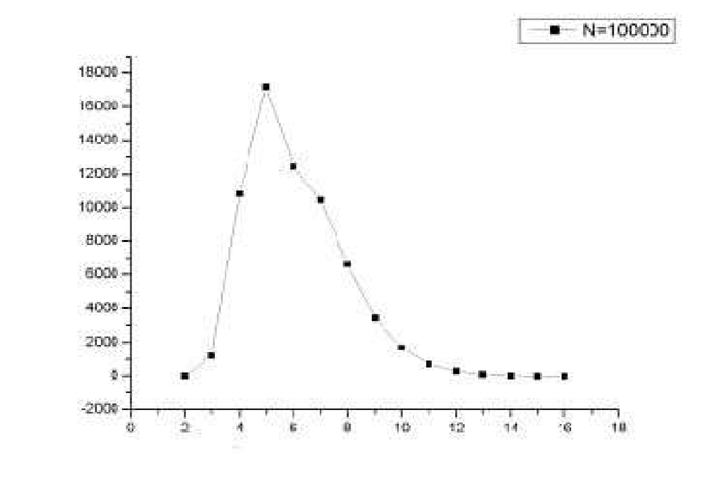
# 20-State Experiment



# 30-State Experiment



# Random Automata

A (partial) explanation of these experimental observations: if $Q$ is an $n$-set (with $n$ large enough), then, on average, any product of $2n$ randomly chosen transformations of $Q$ is a constant map (Higgins, 1988). In automata-theoretic terms, this means that a randomly chosen DFA with $n$ states and a sufficiently large input alphabet tends to be synchronizing and is reset by any word of length $\geq 2n$.

Thus, "slowly" synchronizing automata cannot be discovered via a random sampling.

Synchronizing random automata: what is the expectation of the minimum length of reset words for a random automaton with $n$ states? What is the probability distribution of this length?

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# Greedy Algorithm

**input** $\mathscr{A} = \langle Q, \Sigma, \delta \rangle$ (a DFA)

**initialization** $w \leftarrow 1$ (the empty word)

$P \leftarrow Q$

**while** $|P| > 1$ find a word $v \in \Sigma^*$ of minimum length with $|\delta(P, v)| < |P|$; if none exists, **return** Failure

$w \leftarrow wv$

$P \leftarrow \delta(P, v)$

**return** $w$

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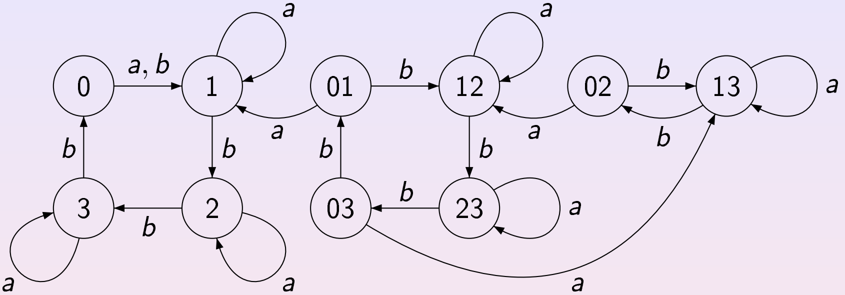


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Observe that the reset word constructed this way is of length 10 while we know a reset word of length 9 for this automaton.

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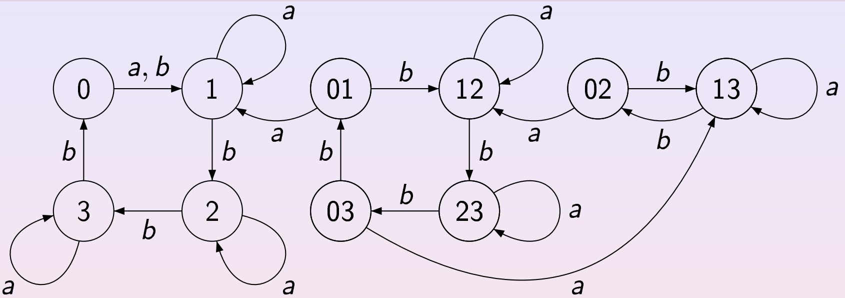


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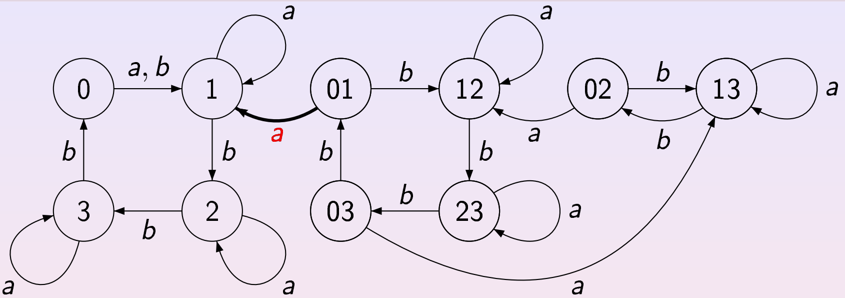


$a, \ Q\,.\,a = \{1, 2, 3\}$ $abba \cdot babbba, \ Q\,.\,abbababbba = \{1\}$

Observe that the reset word constructed this way is of length 10 while we know a reset word of length 9 for this automaton.

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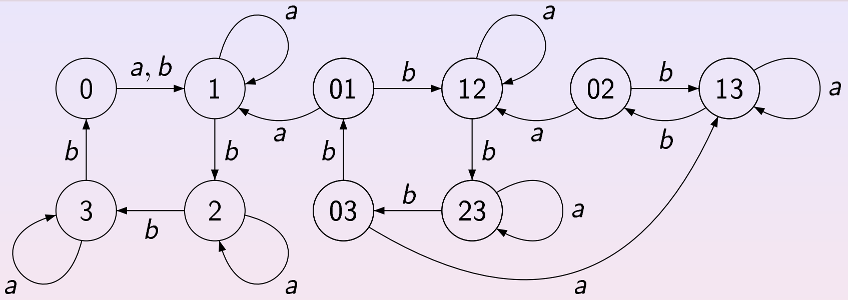


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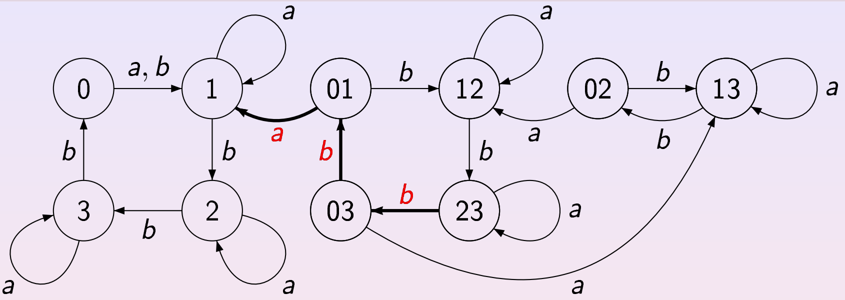


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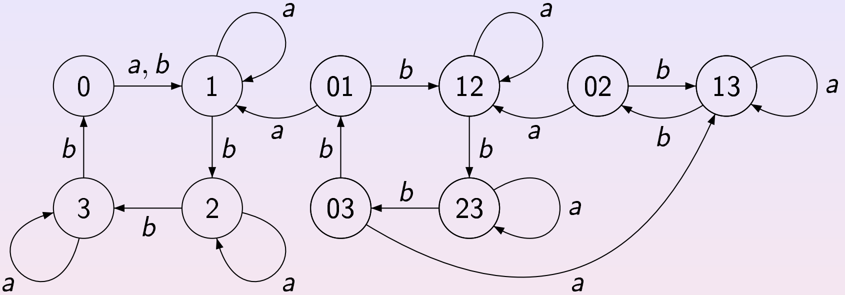


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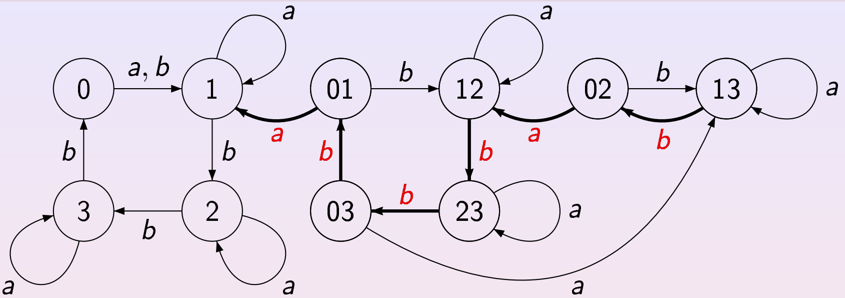


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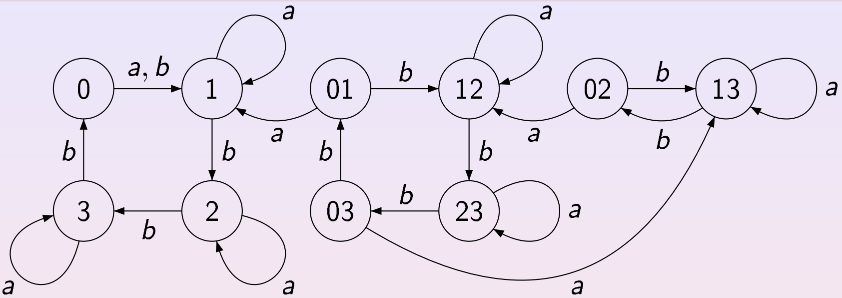


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# Algorithmic Problems

The presently best upper bound for the Černý function for the class of all synchronizing automata ($\frac{n^3-n}{6}$) was achieved by a non-trivial analysis of the greedy algorithm. What is the real potential of the greedy algorithm?

Is it true that, for any synchronizing automaton with $n$ states, the greedy algorithm returns a reset word of length $O(n^2 \log n)$?

Is the same is true for a polynomial algorithm, even non-deterministic?

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# Problems on Synchronizing Digraphs

Let Γ be strongly connected primitive digraph with constant out-degree.

The hybrid Černý/Road Coloring problem: if Γ has $n$ vertices, what is the minimum length of reset words for synchronizing colorings of Γ?

We have the lower bound $n^2 - 3n + 3$ and conjecture that it is tight.

Characterize totally synchronizing digraphs Γ, that is, such that each coloring of Γ is synchronizing. For instance, underlying digraphs of the Černý automata are totally synchronizing, and so are Wielandt's digraphs yielding the lower bound in the previous problem.

SATA, Sept 10th, 2008

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Let $\Gamma$ be strongly connected primitive digraph with constant out-degree.

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Universal reset words for a given digraph: we say that a word $w$ is a universal reset word for $\Gamma$ if it is a reset word for every synchronizing coloring of $\Gamma$. If $\Gamma$ has $n$ vertices, what is the minimum length of universal reset words for $\Gamma$?

Complexity issues, for instance, complexity of calculating short (or shortest) reset words for digraphs. Given $\Gamma$ and a positive integer $n$, is there a reset word of length $n$ for a synchronizing coloring of $\Gamma$? Clearly, the problem is in NP but it may even be in P.

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