Open Problems on Synchronizing Automata

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- Algorithmic and Complexity Problems

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- Problems Dealing with Synchronizing Digraphs

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- Careful Synchronization

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- The Černý function's behaviour for various restricted classes of synchronizing automata.

Aperiodic Automata

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Synchronization issues remain difficult when restricted to the class $\mathbf{Ap}$ of all aperiodic automata.

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Similar things happen for several other classes of synchronizing automata for whose Černý functions we know an upper bound (Eulerian, weakly monotonic).

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2. There are four different states $q_1, q_2, q_3, q_4 \in Q$ such that

$$\delta(q_1, a) = \delta(q_2, a) \neq \delta(q_3, a) = \delta(q_4, a).$$

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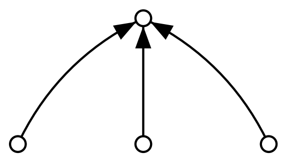
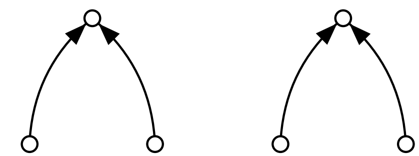


The action of a dromedary letter



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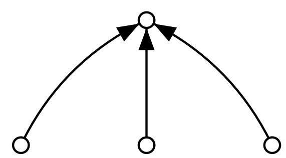

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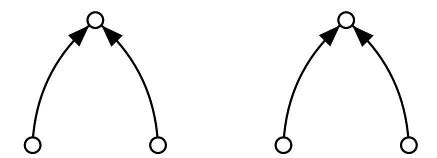

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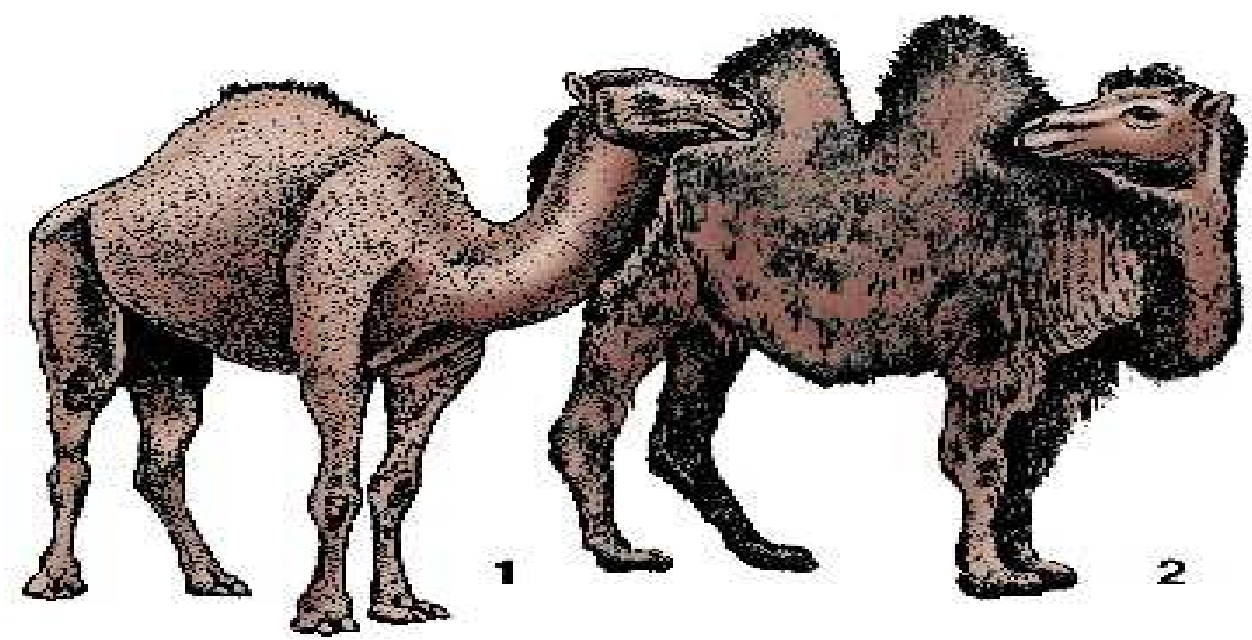

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Do these lower bounds represent the worst possible case? In other words, is the Černý function for the class of synchronizing automata with a dromedary/bactrian letter equal to $(n-2)^2+1$/respectively $(n-1)(n-2)$?

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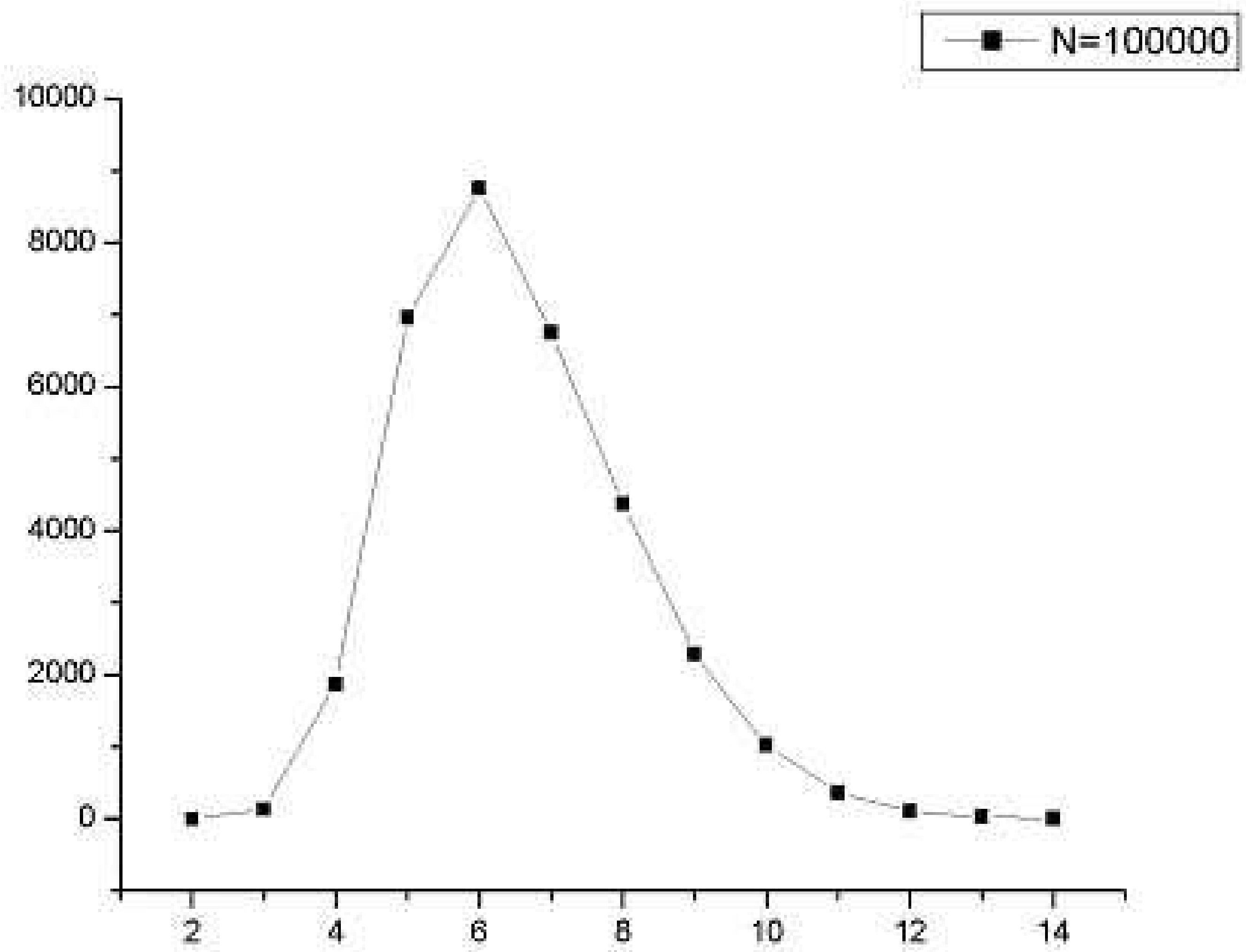
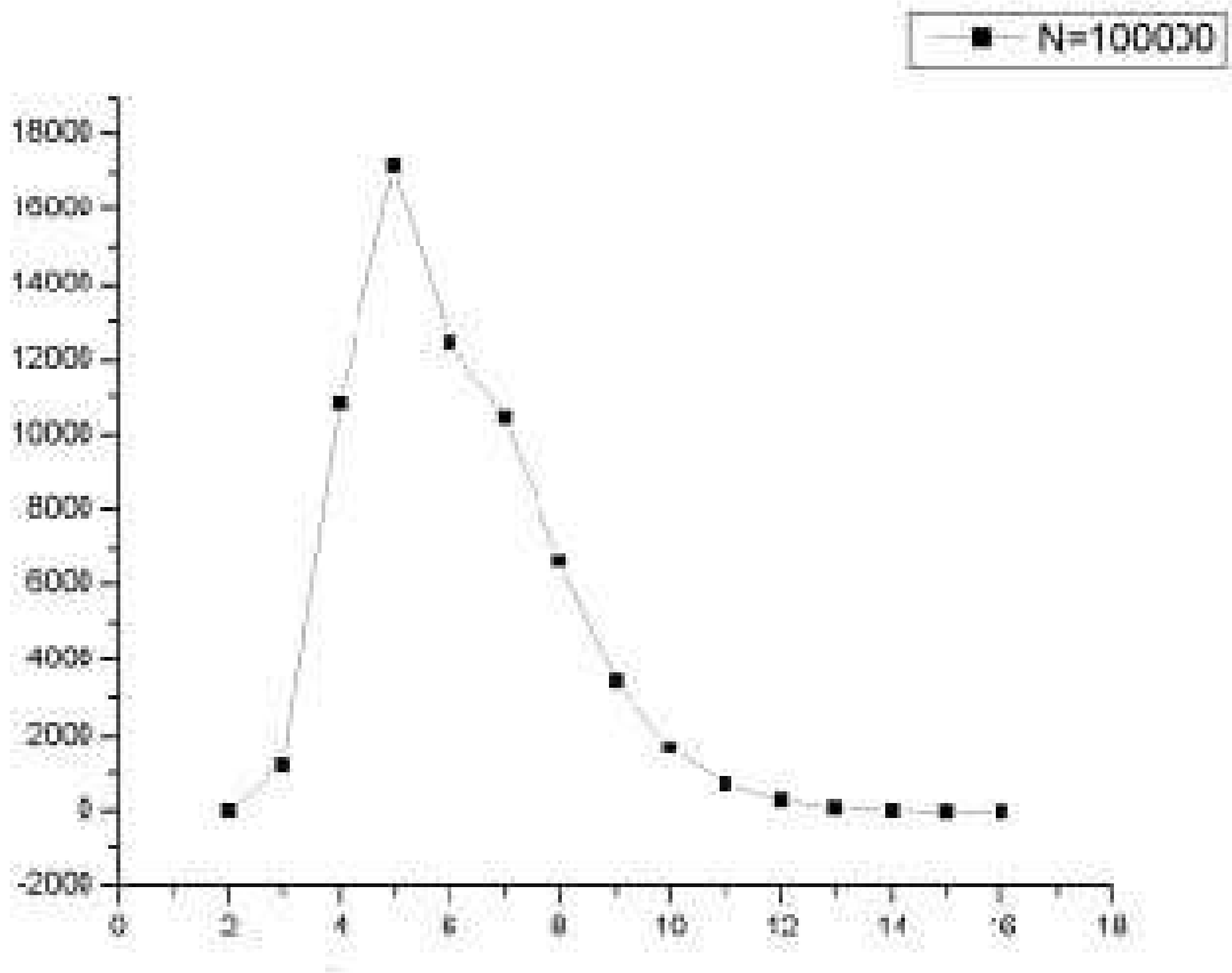
Observe that again the original Černý conjecture correspond to the case $k = 1$.

20-State Experiment



30-State Experiment



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Synchronizing random automata: what is the expectation of the minimum length of reset words for a random automaton with n states? What is the probability distribution of this length?

Greedy Algorithm

input $\mathscr{A} = \langle Q, \Sigma, \delta \rangle$ (a DFA)
initialization $w \leftarrow 1$ (the empty word)

$P \leftarrow Q$
while $|P| > 1$ find a word $v \in \Sigma^*$ of minimum length with $|\delta(P, v)| < |P|$; if none exists, **return** Failure
$w \leftarrow wv$
$P \leftarrow \delta(P, v)$
return w

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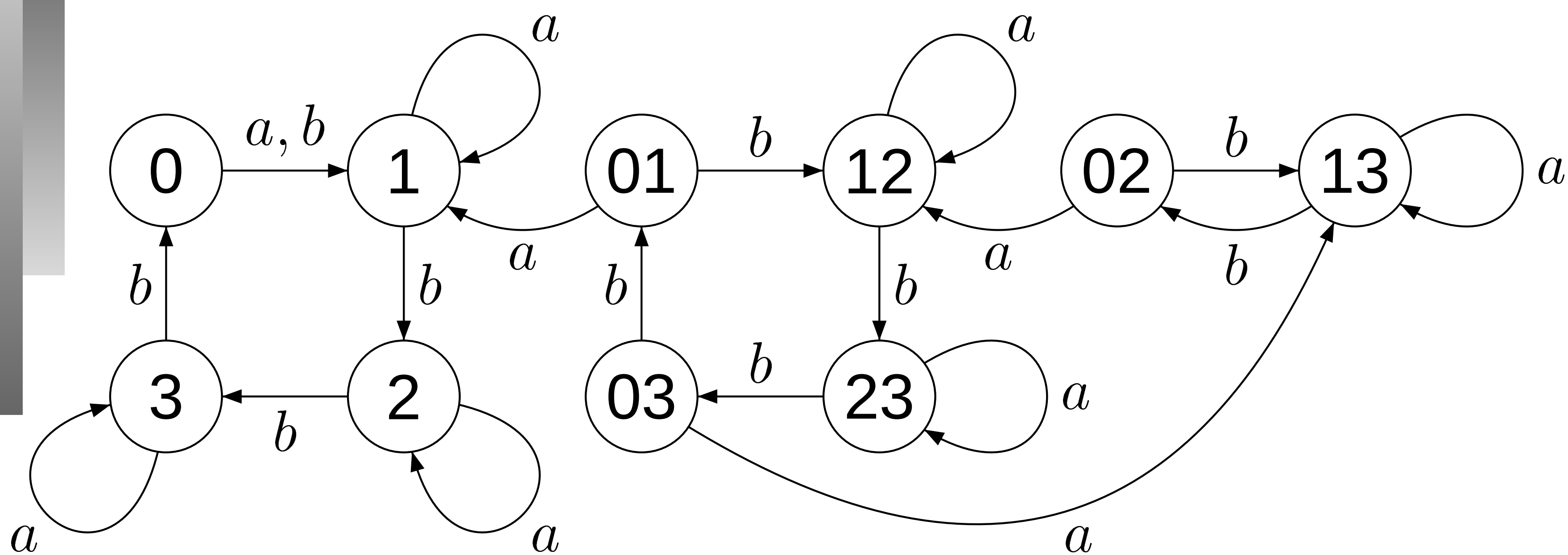

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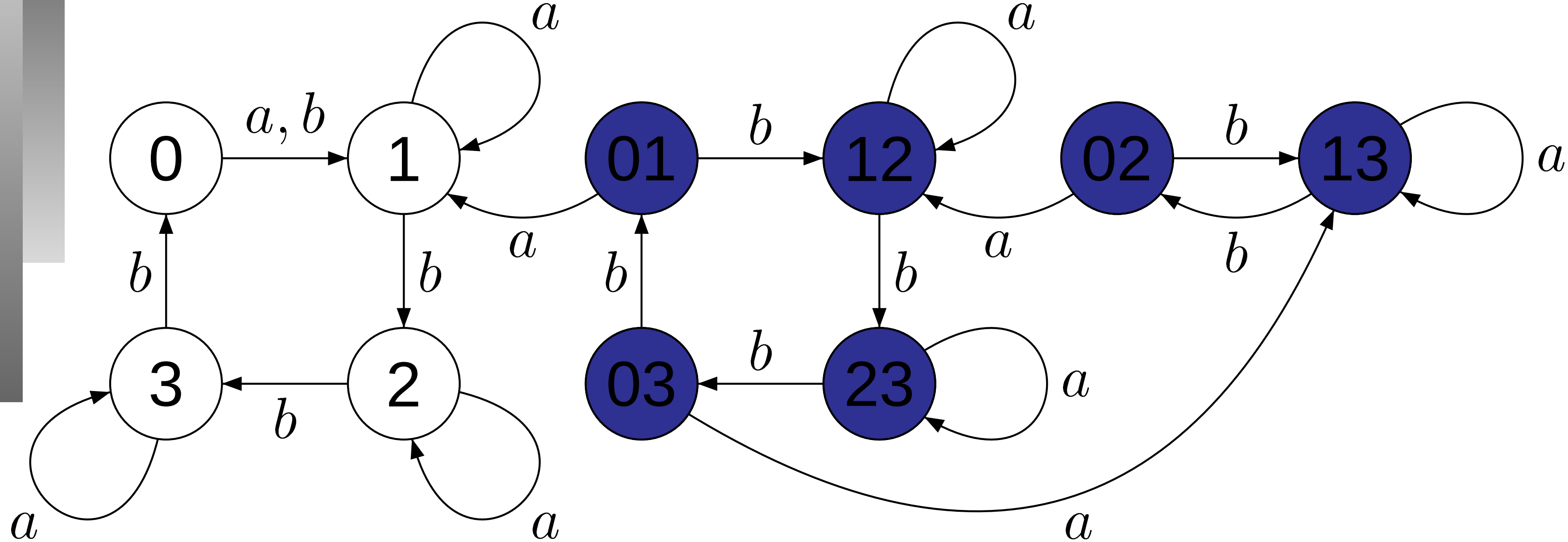

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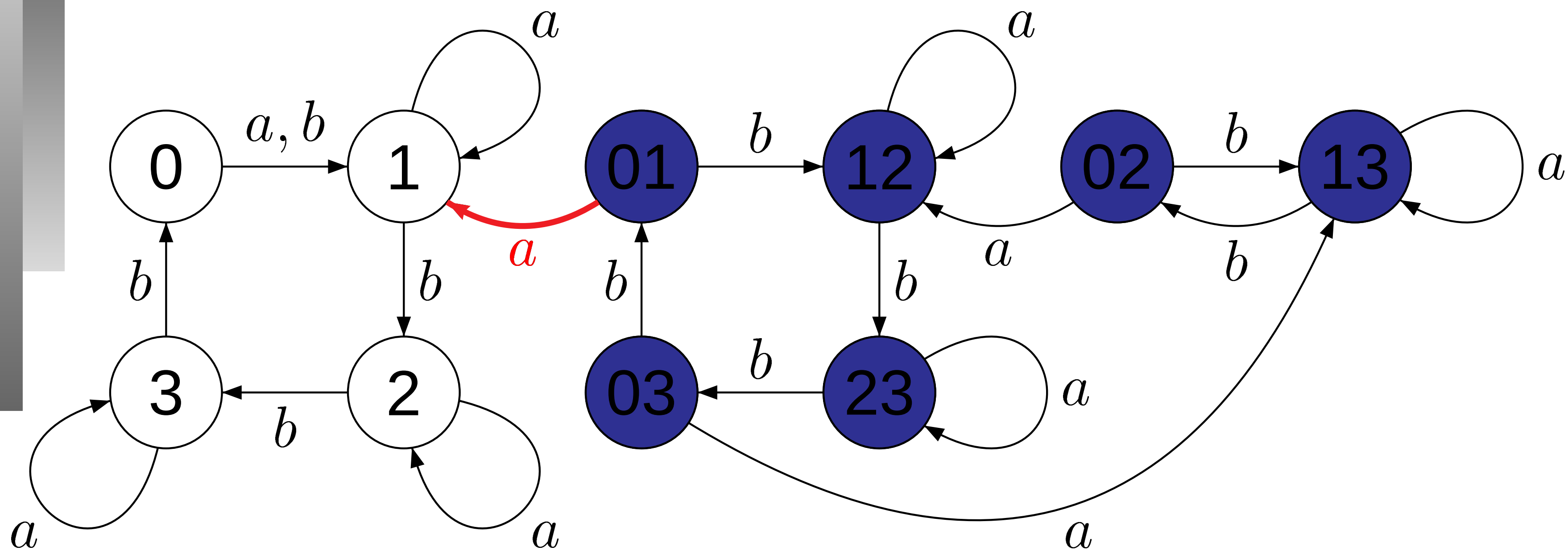

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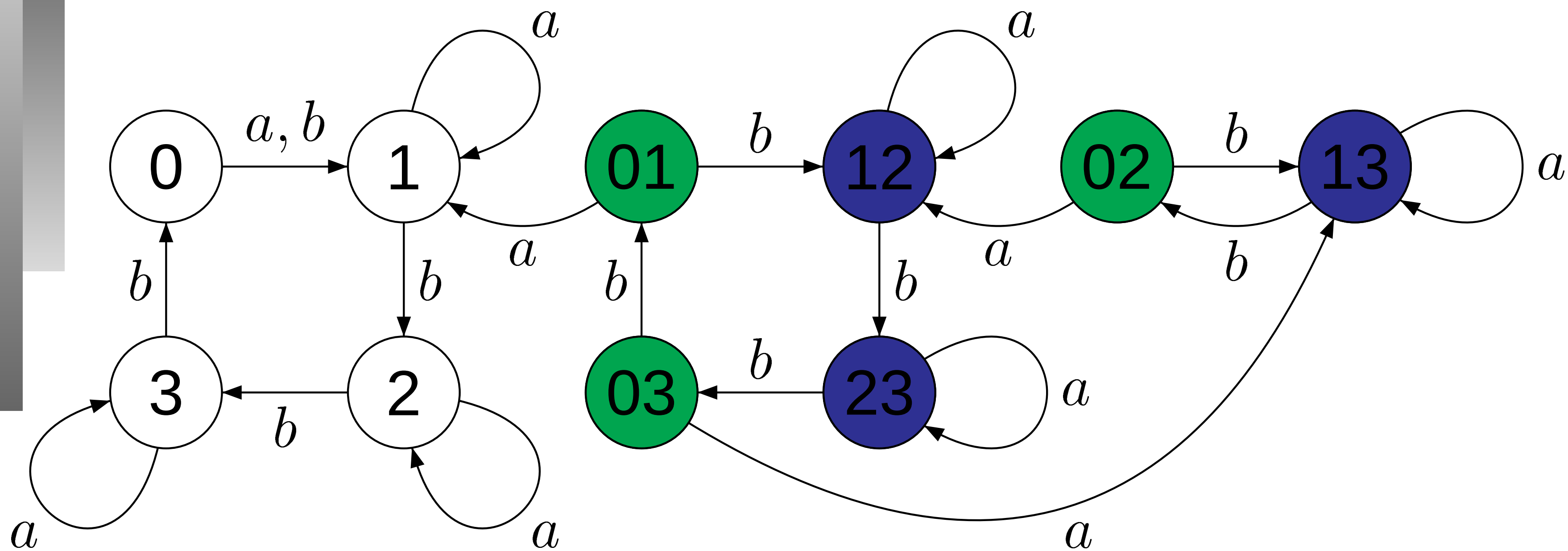

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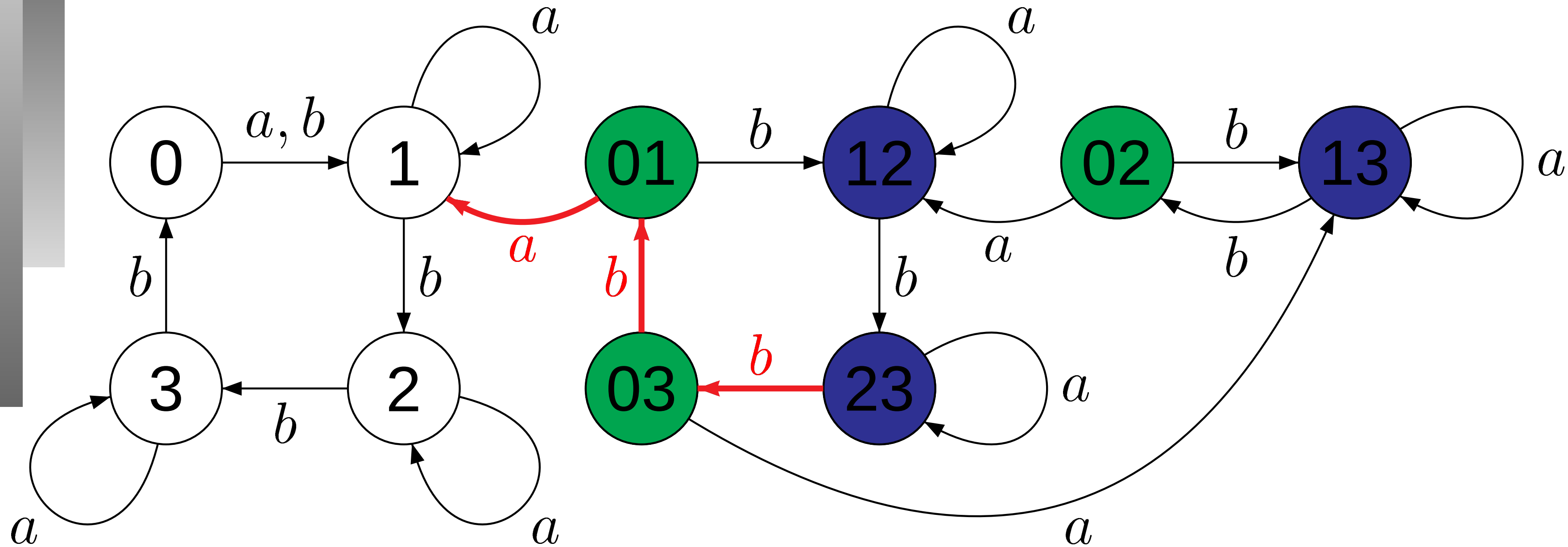

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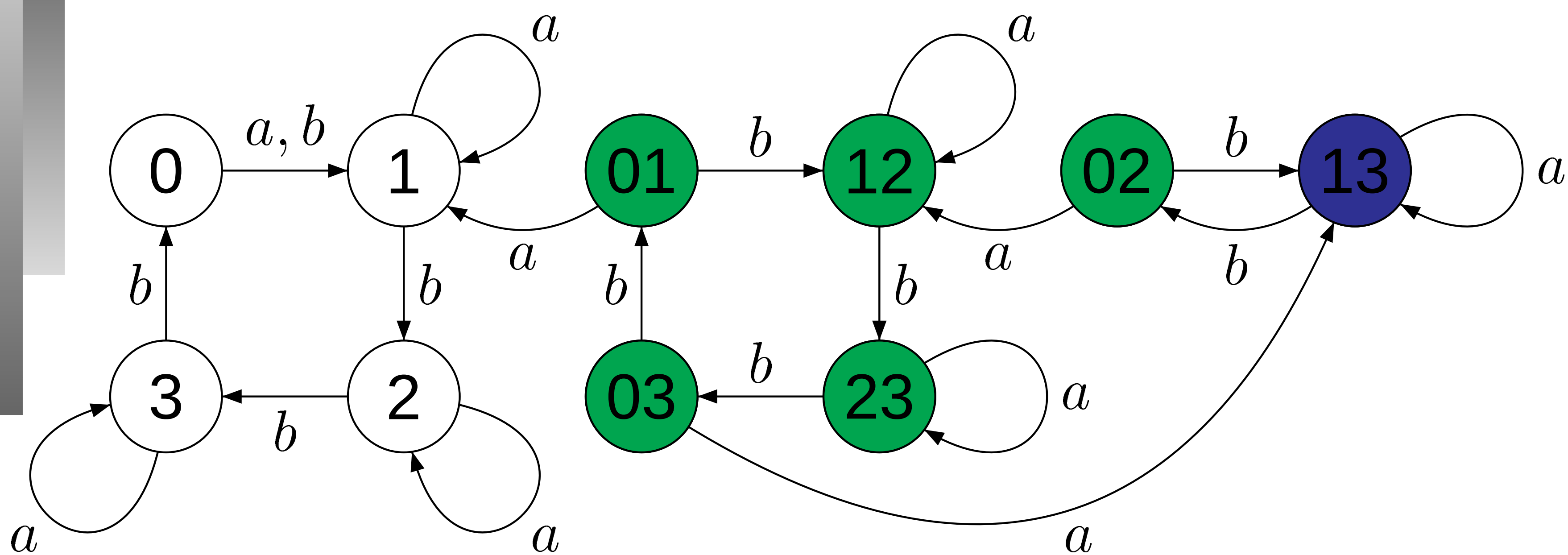

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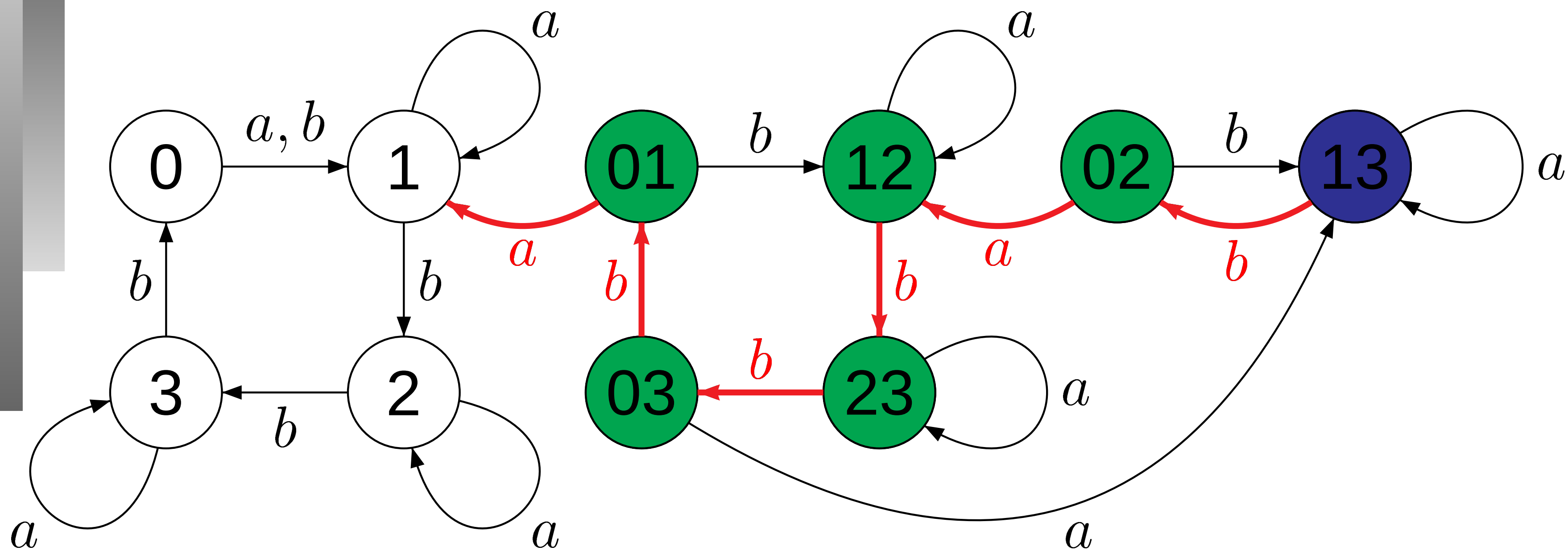

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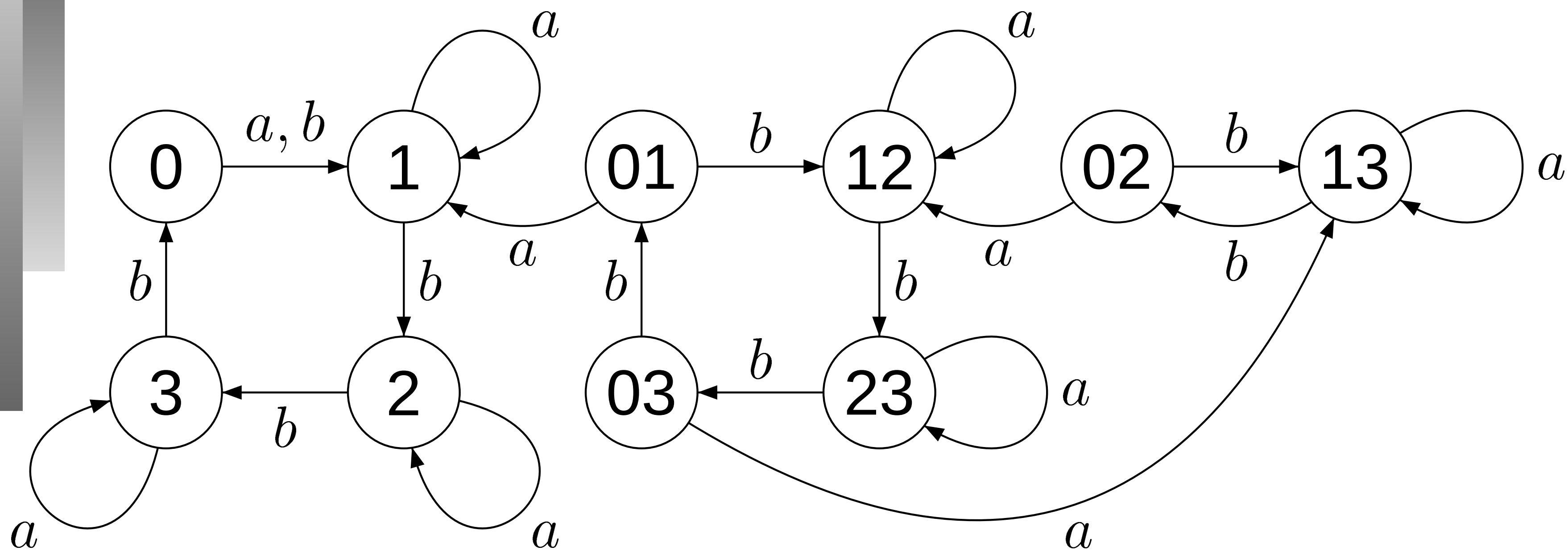

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Observe that the reset word constructed this way is of length 10 while we know a reset word of length 9 for this automaton.

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Is it true that, for any synchronizing automaton with n states, the greedy algorithm returns a reset word of length $\Omega(n^2 \log n)$?

Is the same is true for a polynomial algorithm, even non-deterministic?

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- Characterize totally synchronizing digraphs Γ, that is, such that each coloring of Γ is synchronizing. For instance, underlying digraphs of the Černý automata are totally synchronizing, and so are Wielandt's digraphs yielding the lower bound in the previous problem.

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- Complexity issues, for instance, complexity of calculating short (or shortest) reset words for digraphs. Given Γ and a positive integer n, is there a reset word of length n for a synchronizing coloring of Γ? Clearly, the problem is in NP but it may even be in P.

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  But in the absence of the constant out-degree condition, the resulting automaton $\mathscr{A} = \langle Q, \Sigma, \delta \rangle$ is incomplete. We need a suitable modification of the notion of a synchronizing automaton for this case.

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We say that $w = a_1 \cdots a_\ell$ with $a_1, \ldots, a_\ell \in \Sigma$ is a *careful reset word* for $\mathscr{A}$ if

- $\delta(q, a_1)$ is defined for all $q \in Q$,
- $\delta(q, a_i)$ with $1 < i \leq \ell$ is defined for all

$$q \in Q \,.\, a_1 \cdots a_{i-1},$$

- $|Q \,.\, w| = 1$.

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$$q \in Q\,.\,a_1 \cdots a_{i-1},$$
- $|Q\,.\,w| = 1$.

Careful synchronization has been much studied recently and has proved to be more complicated than the usual synchronization.

Careful Synchronization

We say that $w = a_1 \cdots a_\ell$ with $a_1, \ldots, a_\ell \in \Sigma$ is a *careful reset word* for $\mathscr{A}$ if
- $\delta(q, a_1)$ is defined for all $q \in Q$,
- $\delta(q, a_i)$ with $1 < i \leq \ell$ is defined for all

$$q \in Q \,.\, a_1 \cdots a_{i-1},$$

- $|Q \,.\, w| = 1$.

Careful synchronization has been much studied recently and has proved to be more complicated than the usual synchronization. The minimum length of careful reset words may be exponential of the number of states, checking that an automaton is carefully synchronizing is PSPACE-complete.

Careful Synchronization

In the transport network terms careful synchronization means that there is an instruction which is always possible to follow and which brings one to the node which is independent of the initial node.

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Primitivity is still necessary but it is not sufficient anymore. The next slide presents an example of a strongly connected primitive digraph which admits no carefully synchronizing coloring.

Careful Synchronization