# Synchronizing Finite Automata: an Overview

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# Where I am from?

# Finite Automata

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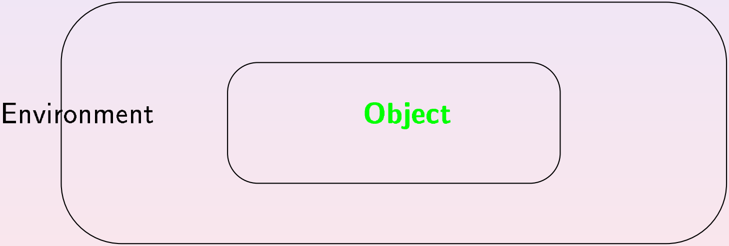

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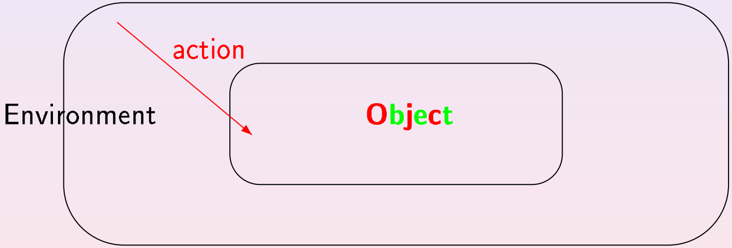

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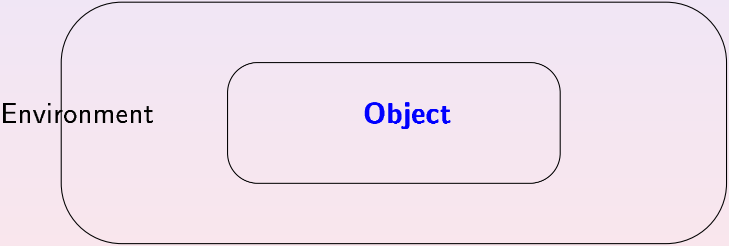

This notion originates in the seminal work by Alan Turing ("On Computable Numbers, With an Application to the Entscheidungsproblem", Proc. London Math. Soc., Ser. 2, 42 (1936), 230–265).

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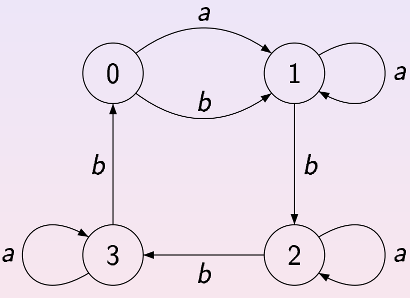

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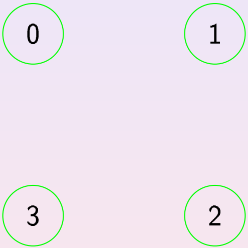


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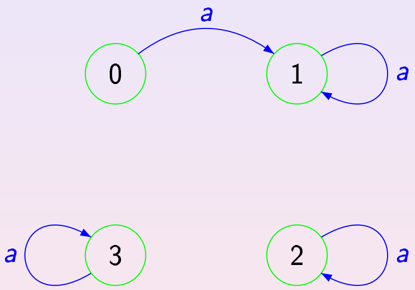


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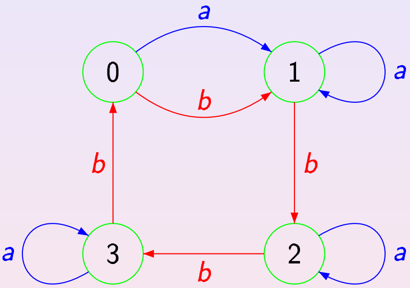


Here one sees 4 **states** called 0,1,2,3, an action called $a$ and another action called $b$.

# Definitions and Terminology

We consider complete deterministic finite automata:

$$\mathscr{A} = \langle Q, \Sigma, \delta \rangle.$$

Here

- $Q$ is the state set;
- $\Sigma$ is the input alphabet;
- $\delta : Q \times \Sigma \to Q$ is the transition function.

We need neither initial nor final states.

$\Sigma^*$ stands for the set of all words over $\Sigma$ including the empty word.

The function $\delta$ uniquely extends to a function $Q \times \Sigma^* \to Q$ still denoted by $\delta$.

To simplify notation we often write $q \,.\, w$ for $\delta(q, w)$ and $P \,.\, w$ for $\{\delta(q, w) \mid q \in P\}$.

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# Definitions and terminology

An automaton $\mathscr{A} = \langle Q, \Sigma, \delta \rangle$ is called synchronizing if there exists a word $w \in \Sigma^*$ whose action resets $\mathscr{A}$, that is, leaves the automaton in one particular state no matter which state in $Q$ it started at: $\delta(q, w) = \delta(q', w)$ for all $q, q' \in Q$.

We can also write this as $|Q \, . \, w| = 1$.

Any word $w$ with this property is a reset word for $\mathscr{A}$.

Other names:

- for automata: directable, cofinal, collapsible, etc;
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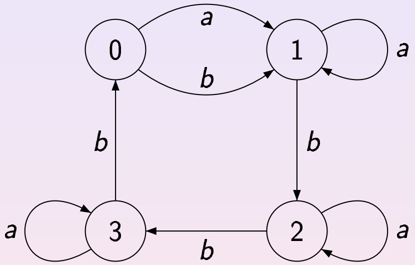


A reset word is *abbbabbba*: applying it at any state brings the automaton to the state 1.

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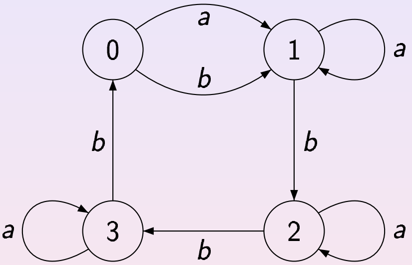


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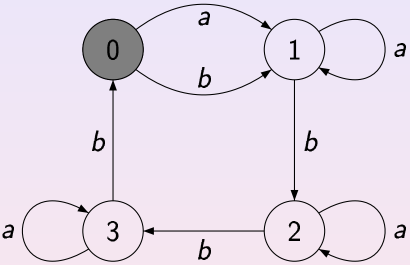


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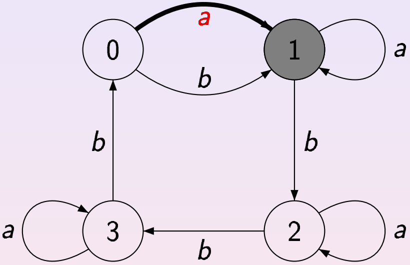


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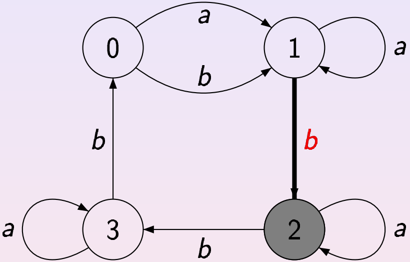


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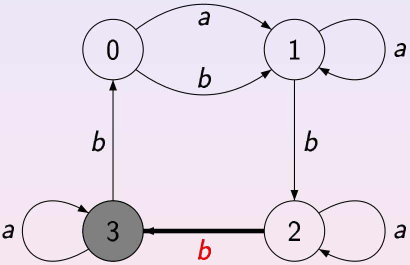


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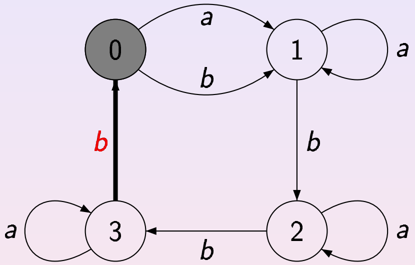


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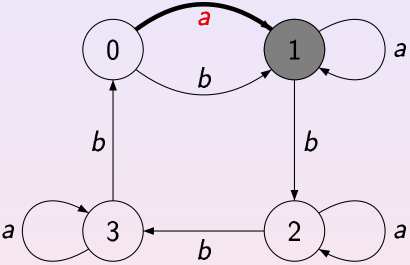


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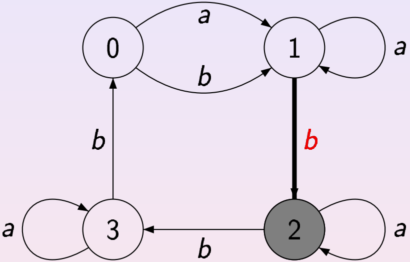


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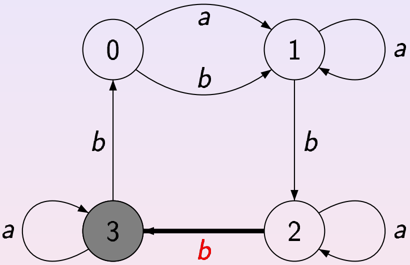


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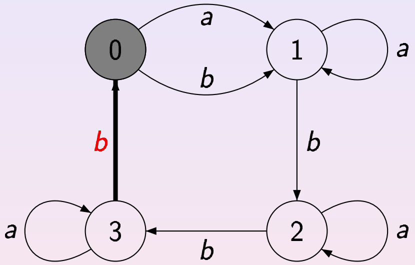


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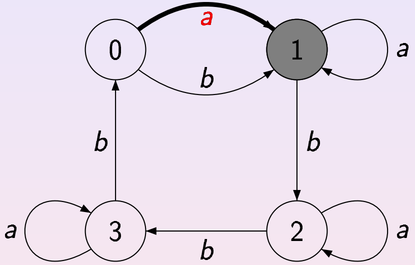


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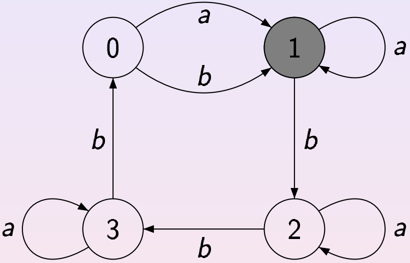


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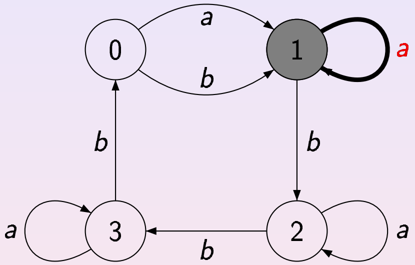


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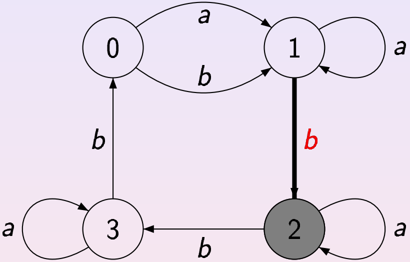


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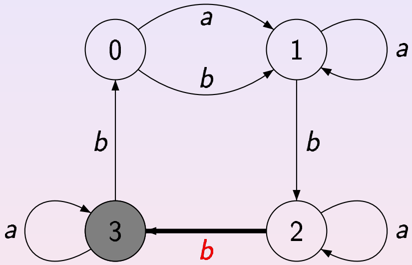


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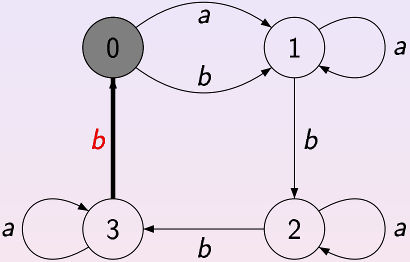


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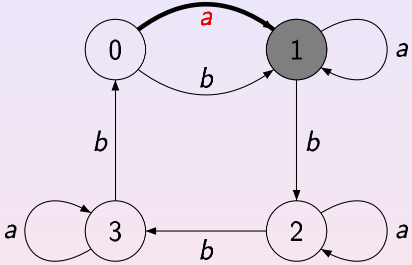


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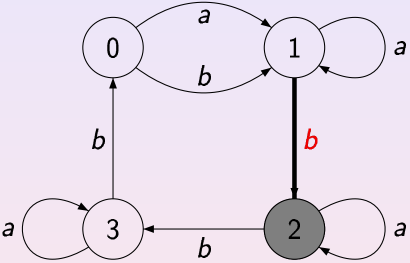


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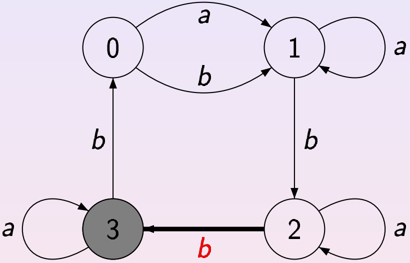


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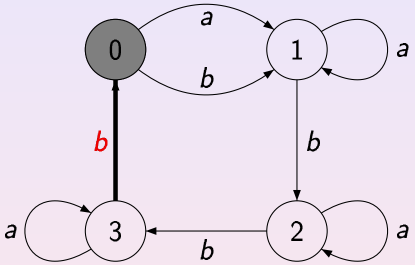


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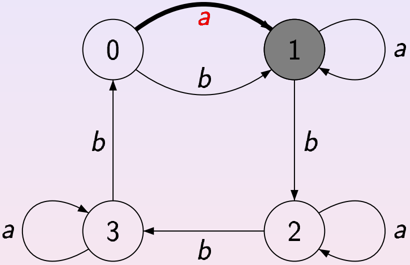


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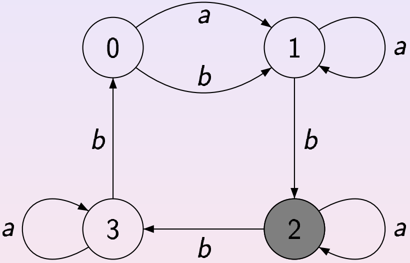


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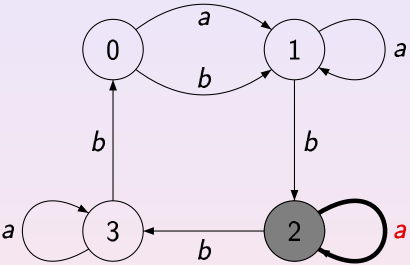


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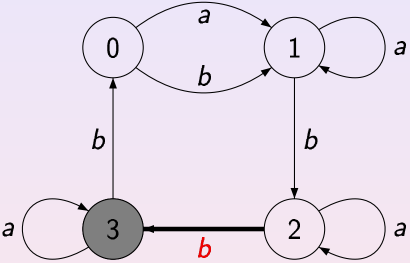


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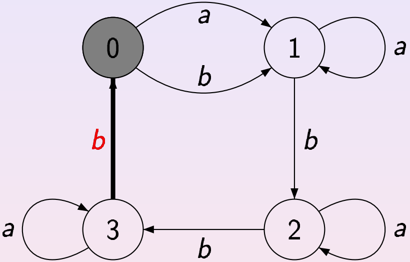


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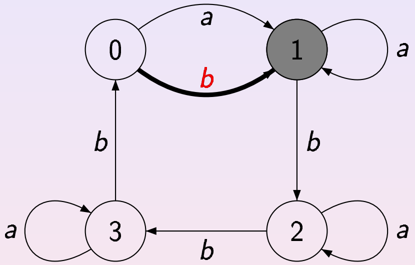


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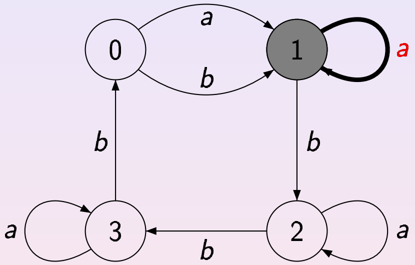


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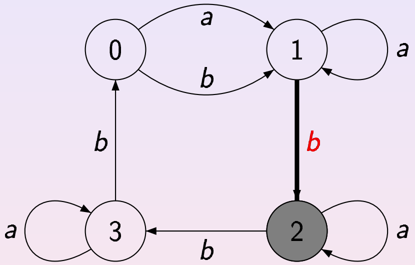


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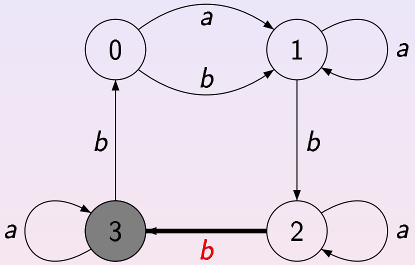


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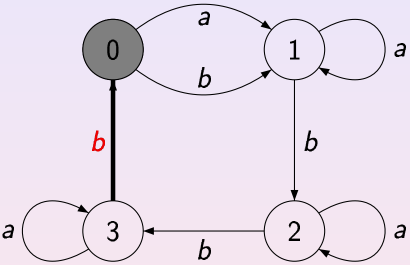


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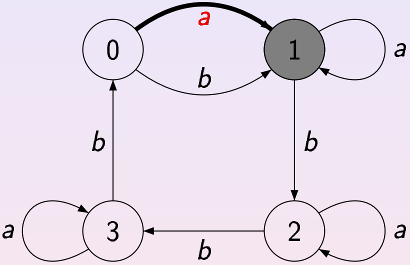


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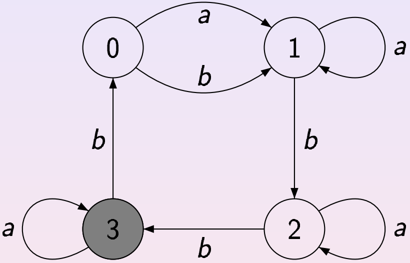


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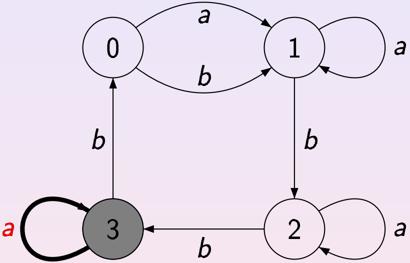


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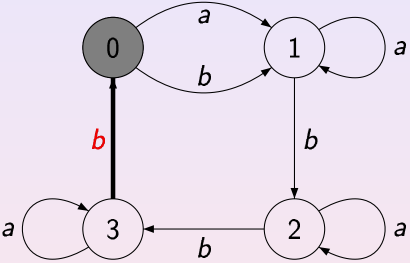


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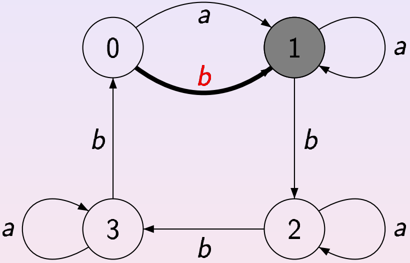


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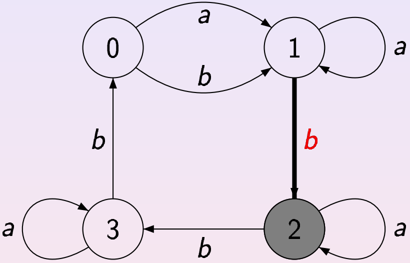


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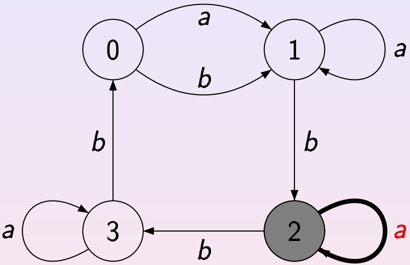


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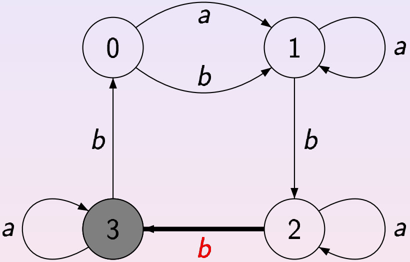


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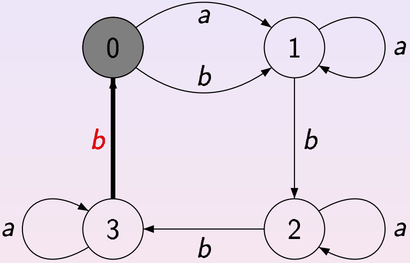


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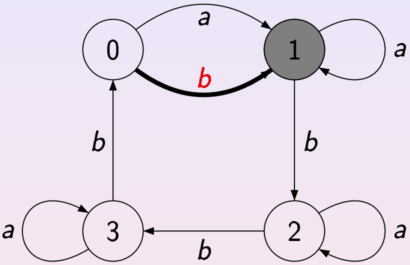


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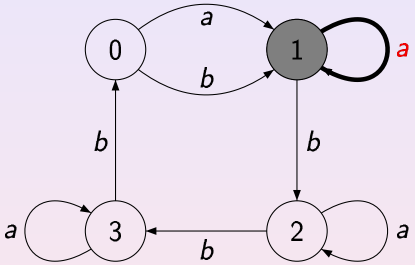


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A prefix code over a finite alphabet $\Sigma$ is a set $X$ of words in $\Sigma^*$ such that no word of $X$ is a prefix of another word of $X$. A prefix code is maximal if it is not contained in another prefix code over the same alphabet. A maximal prefix code $X$ over $\Sigma$ is synchronized if there is a word $x \in X^*$ such that for any word $w \in \Sigma^*$, one has $wx \in X^*$. Such a word $x$ is called a synchronizing word for $X$.

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# Synchronized Codes

$\Sigma = \{0, 1\}$, $X = \{000, 0010, 0011, 010, 0110, 0111, 10, 110, 111\}$.
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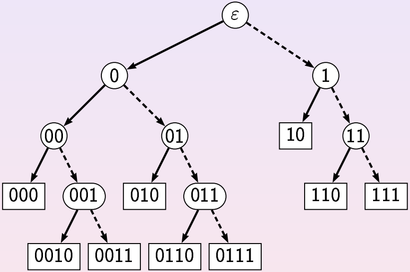


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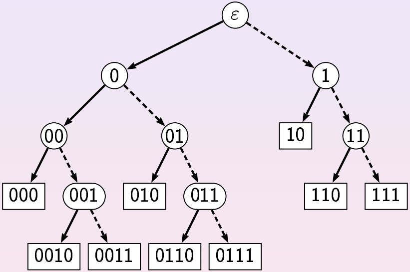


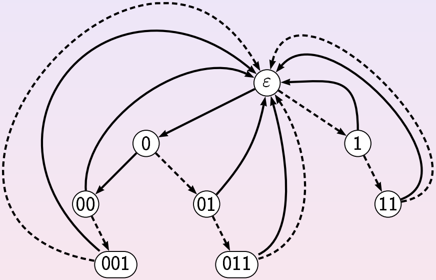


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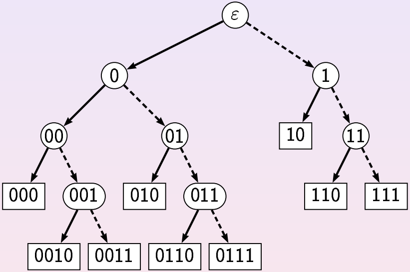


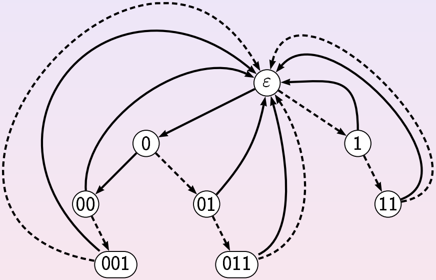


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Since the 60s synchronizing automata have been considered as a useful tool for testing of reactive systems (first circuits, later protocols) and have been also applied in coding theory.

In the 80s, the notion was reinvented by engineers working in a branch of robotics which deals with part handling problems in industrial automation.

Suppose that one of the parts of a certain device has the following shape:



Such parts arrive at manufacturing sites in boxes and they need to be sorted and oriented before assembly.

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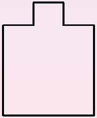

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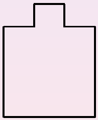

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Assume that only four initial orientations of the part shown above are possible, namely, the following ones:

Suppose that prior the assembly the part should take the 'bump-left' orientation (the second one in the picture). Thus, one has to construct an orienter which action will put the part in the prescribed position independently of its initial orientation.

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We put parts to be oriented on a conveyer belt which takes them to the assembly point and let the stream of the parts encounter a series of passive obstacles of two types (*high* and *low*) placed along the belt.

A high obstacle is high enough so that any part on the belt encounters this obstacle by its rightmost low angle.



Being carried by the belt, the part then is forced to turn 90° clockwise.

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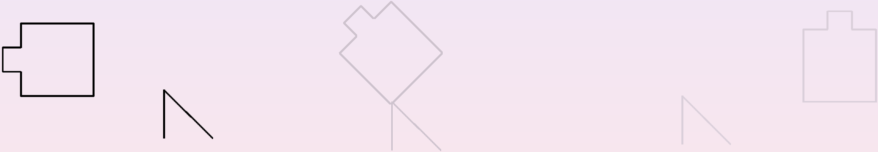

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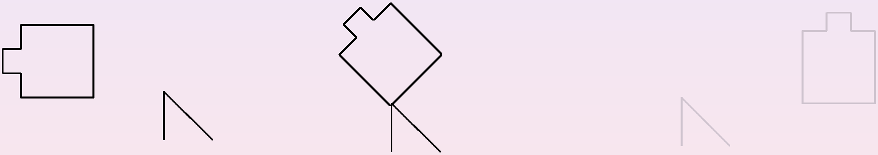

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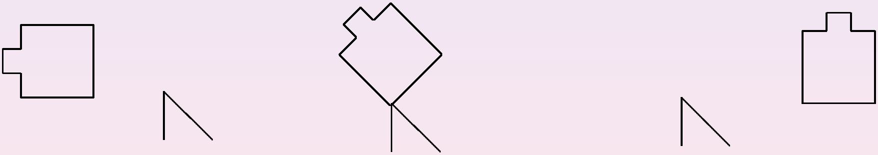

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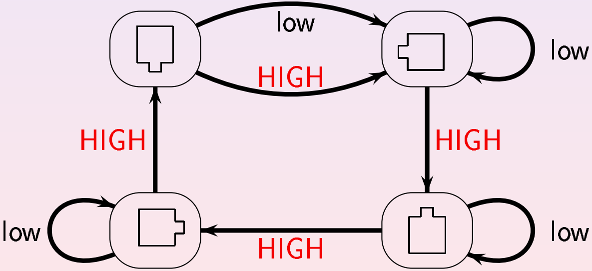

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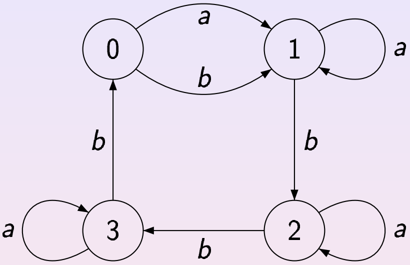


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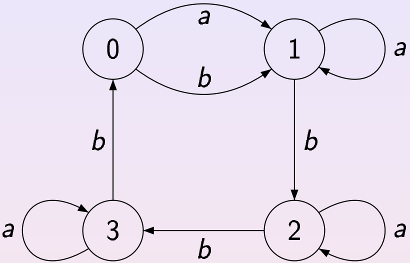


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# Re-inventing by Dynamics Theorists

A substitution on a finite alphabet $X$ is a map $\sigma : X \to X^+$; the substitution is said to be of constant length if all words $\sigma(x)$, $x \in X$, have the same length. One says that $\sigma$ satisfies the coincidence condition if there exist positive integers $m$ and $k$ such that all words $\sigma^k(x)$ have the same letter in the $m$-th position. For an example, consider the substitution $\tau$ on $X = \{0, 1, 2\}$ defined by $0 \mapsto 11,\ 1 \mapsto 12,\ 2 \mapsto 20.$ Calculate the iterations of $\tau$ up to $\tau^4$:

Thus, $\tau$ satisfies the coincidence condition (with $k = 4$, $m = 7$). The coincidence condition completely characterizes the constant length substitutions that give rise to dynamical systems measure-theoretically isomorphic to a translation on a compact Abelian group (Dekking, 1978).

# Re-inventing by Dynamics Theorists

A substitution on a finite alphabet $X$ is a map $\sigma : X \to X^+$; the substitution is said to be of constant length if all words $\sigma(x)$, $x \in X$, have the same length. One says that $\sigma$ satisfies the coincidence condition if there exist positive integers $m$ and $k$ such that all words $\sigma^k(x)$ have the same letter in the $m$-th position. For an example, consider the substitution $\tau$ on $X = \{0, 1, 2\}$ defined by $0 \mapsto 11,\ 1 \mapsto 12,\ 2 \mapsto 20$. Calculate the iterations of $\tau$ up to $\tau^4$:

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There is a straightforward bijection between DFAs and constant length substitutions. Each DFA $\mathscr{A} = \langle Q, \Sigma, \delta \rangle$ with $\Sigma = \{a_1, \dots, a_\ell\}$ defines a length $\ell$ substitution on $Q$ that maps every $q \in Q$ to the word $(q \,.\, a_1) \dots (q \,.\, a_\ell) \in Q^+$.

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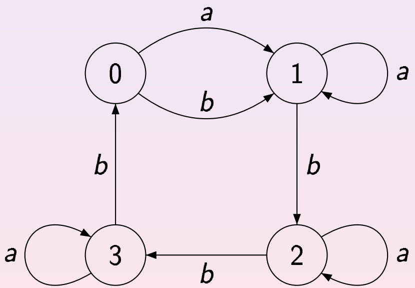


induces the substitution $0 \mapsto 11, \ 1 \mapsto 12, \ 2 \mapsto 23, \ 3 \mapsto 30$.

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Conversely, each substitution $\sigma : X \to X^+$ such that all words $\sigma(x)$, $x \in X$, have the same length $\ell$ gives rise to a DFA for which $X$ is the state set and which has $\ell$ input letters $a_1, \dots, a_\ell$ acting on $X$ as follows: $x\,.\,a_i$ is the symbol in the $i$-th position of the word $\sigma(x)$.

Under this bijection substitutions satisfying the coincidence condition correspond precisely to synchronizing automata, and moreover, given a substitution, the number of iterations at which the coincidence first occurs is equal to the minimum length of reset word for the corresponding automaton.

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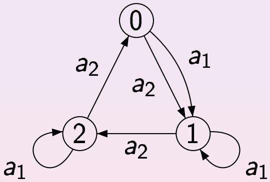


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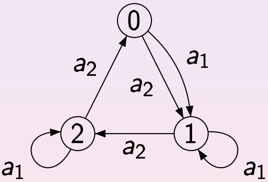


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# Possible Use in Biocomputing

In DNA-computing, there is fast progressing work by Ehud Shapiro's group on "*soup of automata*" (Programmable and autonomous computing machine made of biomolecules, Nature 414, no.1 (November 22, 2001) 430–434; DNA molecule provides a computing machine with both data and fuel, Proc. National Acad. Sci. USA 100 (2003) 2191–2196, etc).

They have produced a solution containing $3 \times 10^{12}$ identical DNA-based automata per $\mu$l. These automata can work in parallel on different inputs (DNA strands), thus ending up in different and unpredictable states. One has to feed the automata with a reset sequence (again encoded by a DNA-strand) in order to get them ready for a new use.

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# Outline of Theory

- From the viewpoint of applications, real or yet imaginary, algorithmic issues are of crucial importance.
- Synchronizing automata constitute an interesting combinatorial object. Their studies from a combinatorial viewpoint are mainly motivated by the Černý Conjecture: every synchronizing automaton with $n$ states has a reset word of length $(n-1)^2$.
- Connections to symbolic dynamics have led to the Road Coloring Problem which has been recently solved by Trahtman.
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