Deterministic expressions and unambiguous languages

Kamal Lodaya

The Institute of Mathematical Sciences, Chennai

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Regular languages

Language L over Σ^*.

- Recognized by automata
- Defined by regular expressions
- Specified by a formula of classical or temporal logic
  - Every a is eventually followed by b
  - $\Box(a \Rightarrow \Diamond b)$
  - $\forall x. \ (a(x) \ \Rightarrow \ \exists y. \ (x < y \land b(y)))$
- Specified algebraically by its syntactic monoid.

Expressiveness Language classes and their corresponding expressions/logics/algebra/automata.

Subclasses of regular languages

Some well known results.

Logic	Interval TL	TL	Reg. Exp.	Automata	Algebraic
$MSO[<]$	QDDC	QPTL	RE	DFA	Monoids
$FO[<]$, $FO^3[<]$	ITL	LTL	SF	CFA	Aperiodic Monoids
$FO^2[<]$	?	UTL	?	$po2dfa$	DA

- LTL $\stackrel{\text{def}}{=}$ TL[U,S,X,Y] (Manna, Pnueli)
- UTL $\stackrel{\text{def}}{=}$ TL[$\Diamond$,$\diamondminus$] (Etessami, Vardi, Wilke)

Answer A fragment of ITL/SF with left- and right-deterministic marked concatenation which we call Unambiguous interval temporal logic/ Unambiguous starfree expressions

Questions

- Membership for regular expressions is NLogSpace-complete
- Membership for starfree expressions is PSpace-complete (Petersen)
- Equivalence for regular expressions is PSpace-complete
- Equivalence for starfree expressions is nonelementary (Stockmeyer and Meyer)
- What about membership and equivalence for unambiguous starfree expressions?

Logic ITL

- Word $w \in \Sigma^+$
- $pos(w)$ set of positions in the word.
- $INTV(w) \stackrel{\text{def}}{=} \{[i,j] \in pos(w)^2 \mid i \leq j\}$
- Satisfaction $w, [i,j] \models D$
- $w \models D$ iff $w, [1, \#w] \models D$.
  $L(D) = \{w \in \Sigma^+ \mid w \models D\}$

Syntax and semantics

Let $a \in \Sigma$. Let D, D_1, D_2 range over formulas in ITL.

Abstract syntax of ITL:

$$pt \;\mid\; \lceil a \rceil \;\mid\; D_1 ; D_2 \;\mid\; D_1 \vee D_2 \;\mid\; \neg D$$

Semantics of ITL:

$w, [i,j] \models pt$ iff $i = j$

$w, [i,j] \models \lceil a \rceil$ iff for all $k : i \leq k \leq j : w[i] = a$

$w, [i,j] \models D_1 ; D_2$ iff for some $k : i \leq k \leq j$ and $w, [i,k] \models D_1$ and $w, [k,j] \models D_2$

Questions

- Modelchecking for ITL is PSpace-complete
- Satisfiability for QDDC and ITL is nonelementary (Stockmeyer and Meyer)
- What about modelchecking and satisfiability for $UITL$?

Results

Reg. Exp.	Membership	Emptiness	Interval TL	Modelchecking	Satisfiability
RE	NLogSpace	PSpace	QDDC	PSpace	Nonelementary
SF	PSpace	Nonelem.	ITL	PSpace	Nonelementary
USF	LogDCFL	PSpace	UITL	Ptime	PSpace

- UTL satisfiability is NP, $FO^2[<]$ satisfiability is NExpTime
- ITL/SF subclasses expressively equivalent to $po2dfa$
- Direct effective translations to $po2dfa$(unlike logics $FO^2[<]$, UTL)

Unambiguous languages

- Studied by Schützenberger (1976)
- A concatenation e_1e_2 is *unambiguous* if for any word w in $L(e_1e_2)$, there is a unique factorization $w = uv$ such that $u \in L(e_1)$ and $v \in L(e_2)$
- An unambiguous language is recognized by a finite monoid which satisfies for some n the equation $(xyz)^{2n} = (xyz)^n y (xyz)^n$
- Such monoids are said to belong to the pseudovariety DA
- Conversely, every monoid in DA recognizes an unambiguous language
- Unambiguous languages are precisely those definable in $FO^2[<]$ (Thérien and Wilke 1998)

Main results

- Unambiguous interval temporal logic (UITL)/ Unambiguous starfree expressions (USF)
- Unique parsability
- Membership in LogDCFL, emptiness in PSpace (NP over a fixed alphabet)
- Effective construction $A: \ UITL \rightarrow po2dfa$ such that $L(D) = L(A(D))$. The size $|A(D)| \ = \ \mathcal{O}(|D|^2)$.
- Effective construction $D: \ po2dfa \rightarrow UITL$ such that $L(A) = L(A(D))$. The size $|D(A)| \ = \ \mathcal{O}(2^{|A|})$.

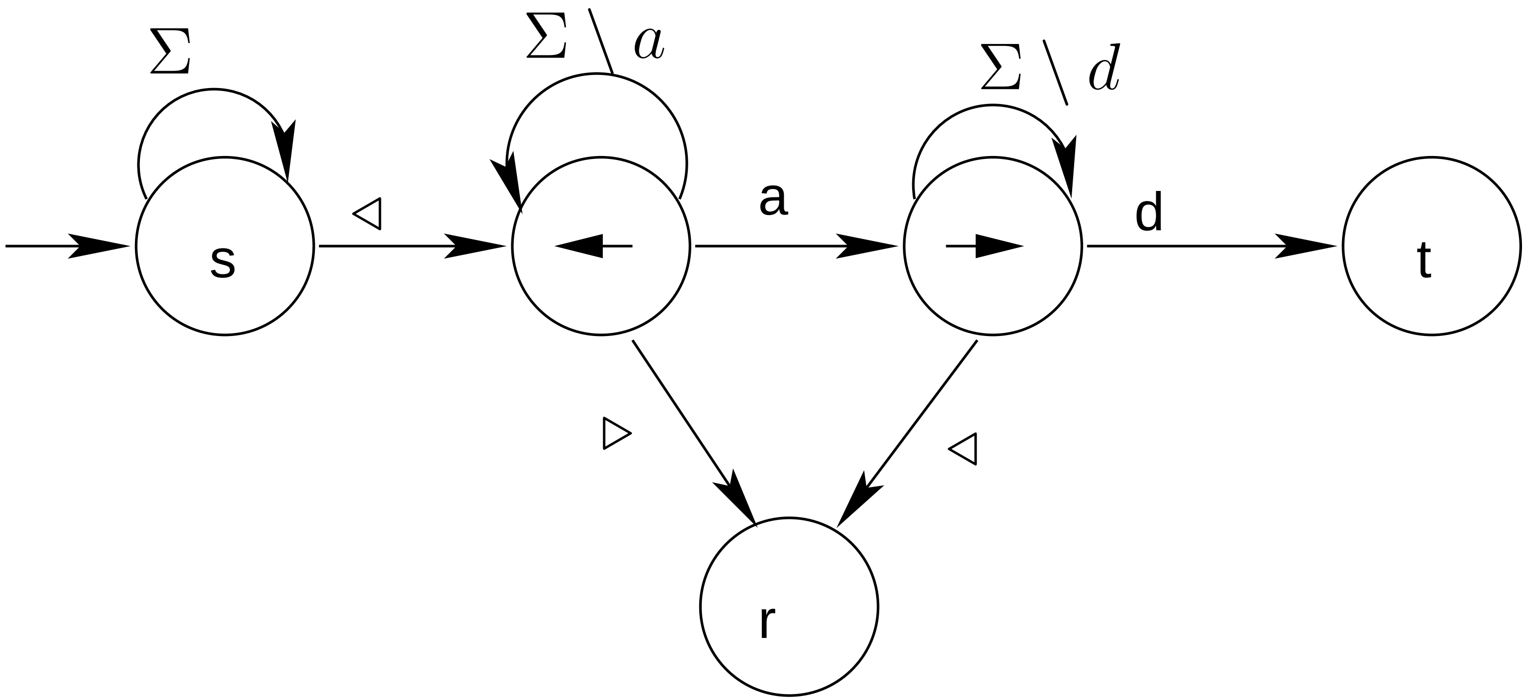
Partially ordered two-way DFA

Introduced by Schwentick, Thérien and Vollmer (2001).
A $po2dfa\ M = (Q, \leq, \Sigma, \delta, s, t, r)$ where

- $(Q, \leq)$ is a poset of states such that r (reject) and t (accept) are the only minimal elements.
- $\delta(q, a) \leq q$ (once a state is exited, it is not re-entered)
- $Q \ = \ Q_l \cup Q_r$ (left- and right-entered)
- $\delta : ((Q_l \cup Q_r) \times \Sigma) \to Q) \cup ((Q_l \times \{\triangleleft\}) \to Q \setminus Q_r) \cup ((Q_r \times \{\triangleright\}) \to Q \setminus Q_l)$
  (endmarkers to prevent falling off the end)

Example automaton

A $po2dfa$ accepting $\Sigma^* a(b+c)^* d(b+c+d)^*$.



Example automaton

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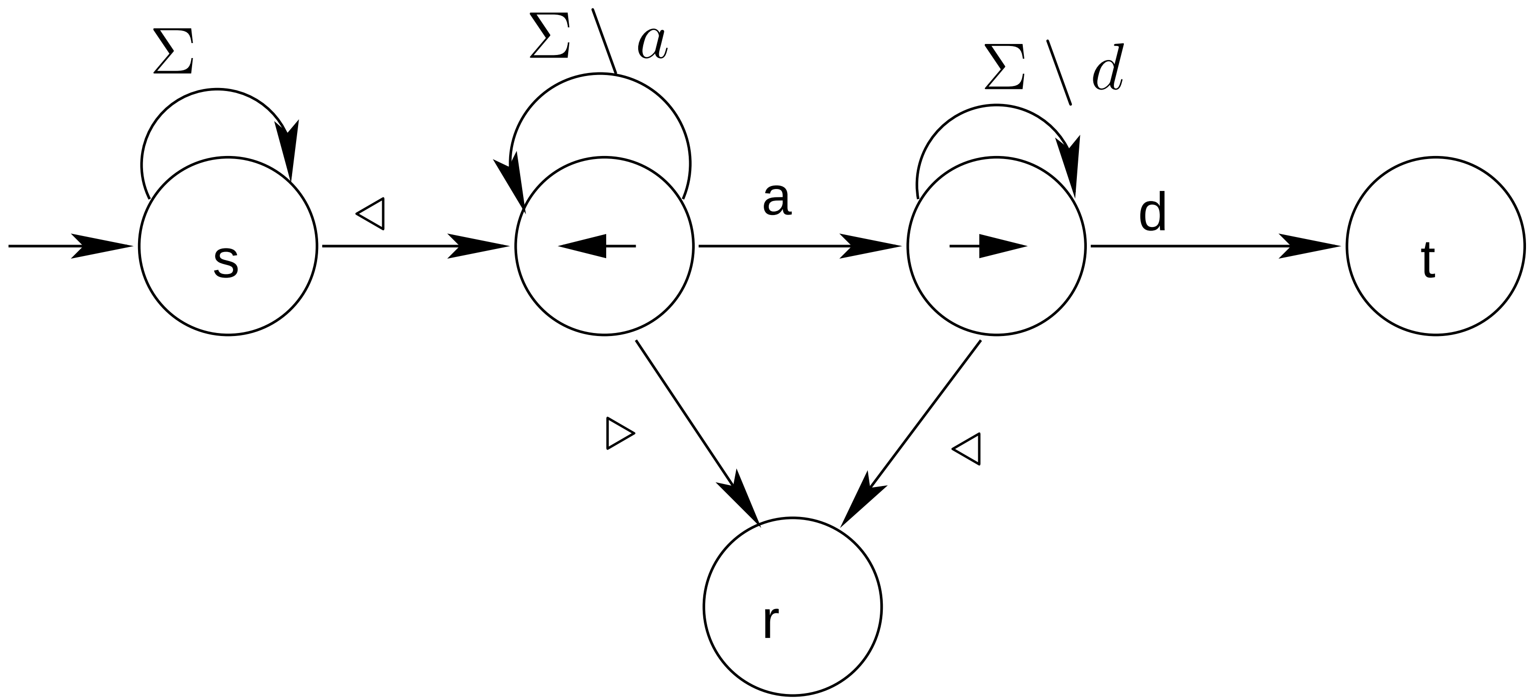
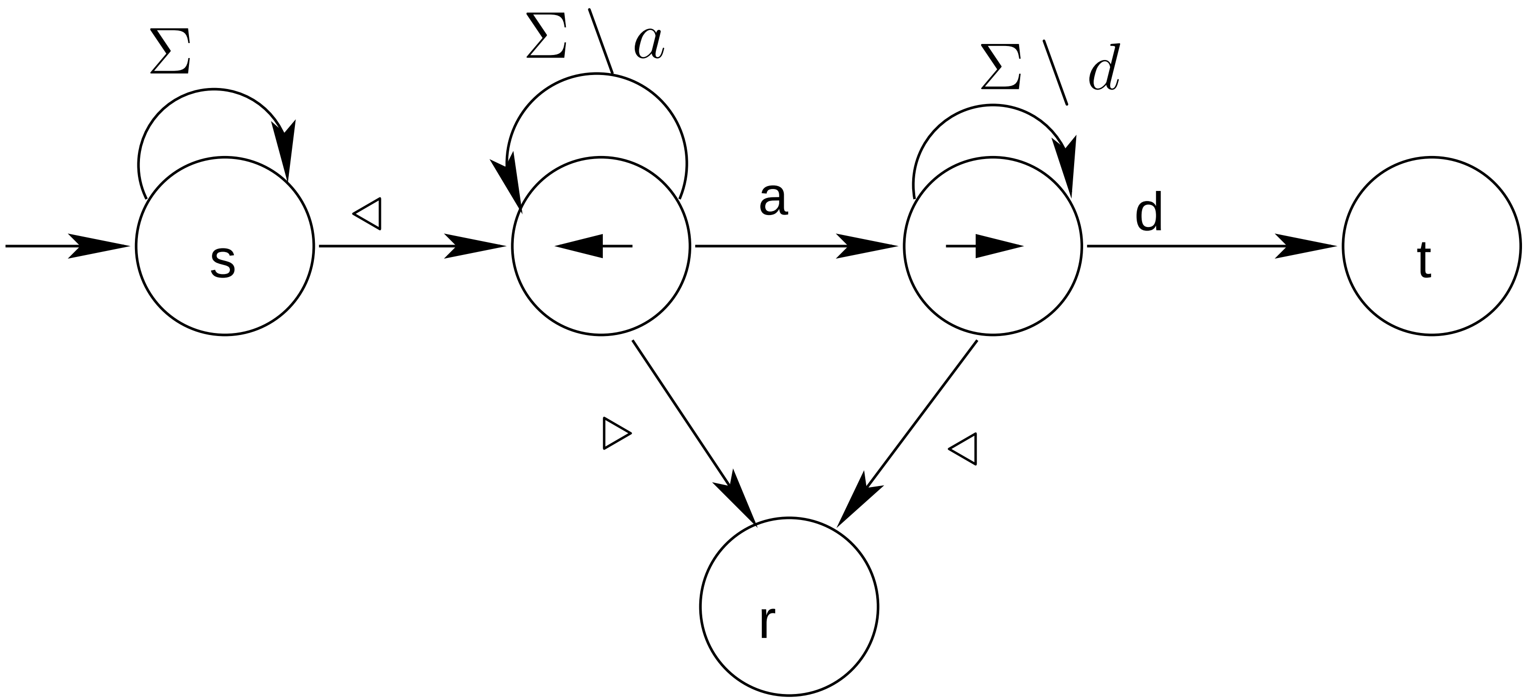


Example Let $w' = \rhd abaab\textcolor{red}{d}dcb\lhd$. Then,
$M(w,*) = (t,6)$ (startable from anywhere).

Example automaton

A $po2dfa$ accepting $\Sigma^* a(b+c)^* d(b+c+d)^*$.



Turtle expression $\xrightarrow{\triangleleft}; \;\; \xleftarrow{a}; \;\; \xrightarrow{d}$

Run of $po2dfa$

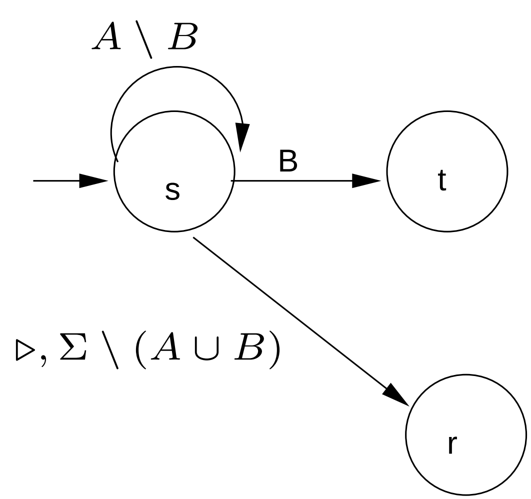
Given input w a $po2dfa$ $M = (Q, \leq, \delta, s, t, r)$ operates as follows:

- Scans the word $w' = \triangleright w \triangleleft$ in both directions.
- Configuration (q, i)
- $(q, i) \rightarrow (q', i')$ if $q, a, q' \in \delta$ and $w[i] = a$. Also, $i' = i + 1$ if $q' \in Q_l$ and $i' = i - 1$ if $q' \in Q_r$. We have $i' = i$ if $q' = t$ or $q' = r$,
- A run from position p_0 is a sequence of transitions $(q_0, p_0), (q_1, p_1), ...(q_f, p_f)$ with $q_0 = s$ and $q_f \in \{t, r\}$.
- $M(w, p_0) \ = \ (q_f, p_f)$.

Extended turtle expressions

Compositional way of constructing $po2dfa$, extending the turtle expressions originally defined by Schwentick, Thérien and Vollmer (2001).

- Acc accepts and Rej rejects without moving the head.
- Given $A, B \subseteq \Sigma'$, let $A \xrightarrow{B}$ be



- $A \xleftarrow{B}$

Extended turtle expressions

- $P?Q, R$ executes P first.
  If P accepts at position j then Q is executed from head position j.
  - Otherwise R is executed from head position j.
- $P;Q \stackrel{\text{def}}{=} P?Q, Rej$ and $\neg P \stackrel{\text{def}}{=} P?Rej, Acc$.

Unambiguous interval temporal logic

Property Between the last a and subsequent first d there must be no b.

Language Let $\Sigma = \{a, b, c, d\}$

$$\Sigma^* a c^* d \{b, c, d\}^*$$

Formula $D \stackrel{\text{def}}{=} (\top L_a ((\neg(\top F_b \top)) F_d \top))$

Logic UITL

Let $a \in \Sigma$. Let D, D_1, D_2 range over formulas in $UITL$.
(We use $\top$ for "true").

Abstract syntax of $UITL$:

$$\top \mid pt \mid D_1 F_a D_2 \mid D_1 L_a D_2 \mid D_1 \vee D_2 \mid \neg D$$
$$\mid \oplus D \mid \ominus D$$

Semantics

$w, [i, j] \models pt$ iff $i = j$

$w, [i, j] \models D_1 F_a D_2$ iff for some $k : i \leq k \leq j.\quad w[k] = a$ and
$\quad$ (for all $m : i \leq m < k.\ w[m] \neq a$) and
$\quad$ $w, [i, k] \models D_1$ and $w, [k, j] \models D_2$

$w, [i, j] \models D_1 L_a D_2$ iff for some $k : i \leq k \leq j.\quad w[k] = a$ and
$\quad$ (for all $m : k < m \leq j.\ w[m] \neq a$) and
$\quad$ $w, [i, k] \models D_1$ and $w, [k, j] \models D_2$

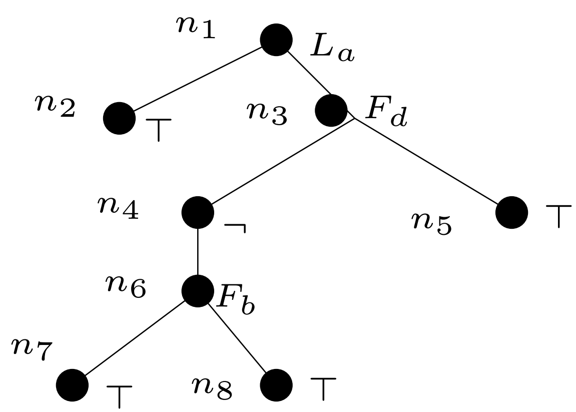
$w, [i, j] \models \oplus D$ iff $i < j$ and $w, [i + 1, j] \models D$

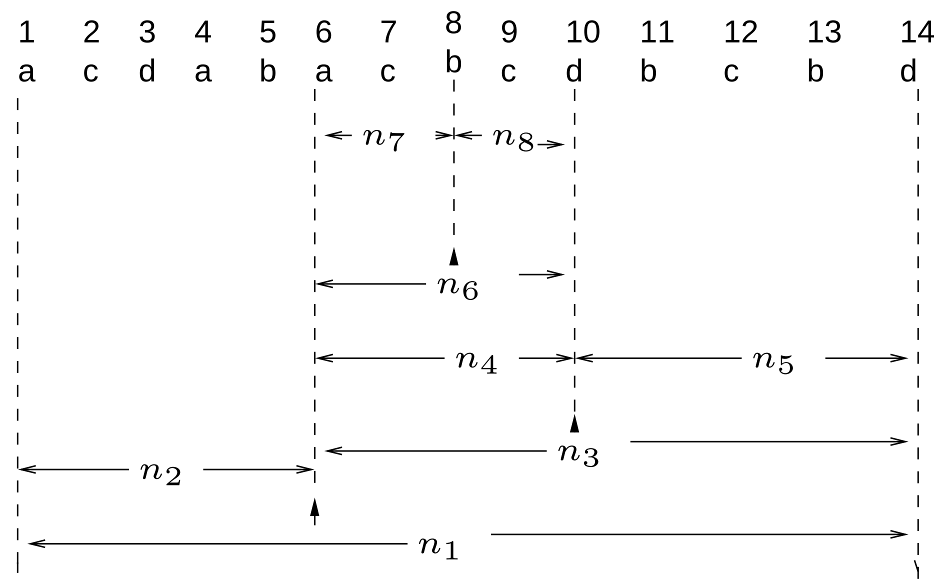
$w, [i, j] \models \ominus D$ iff $i < j$ and $w, [i, j - 1] \models D$

Note: $\lceil a \rceil$ is definable.

Example

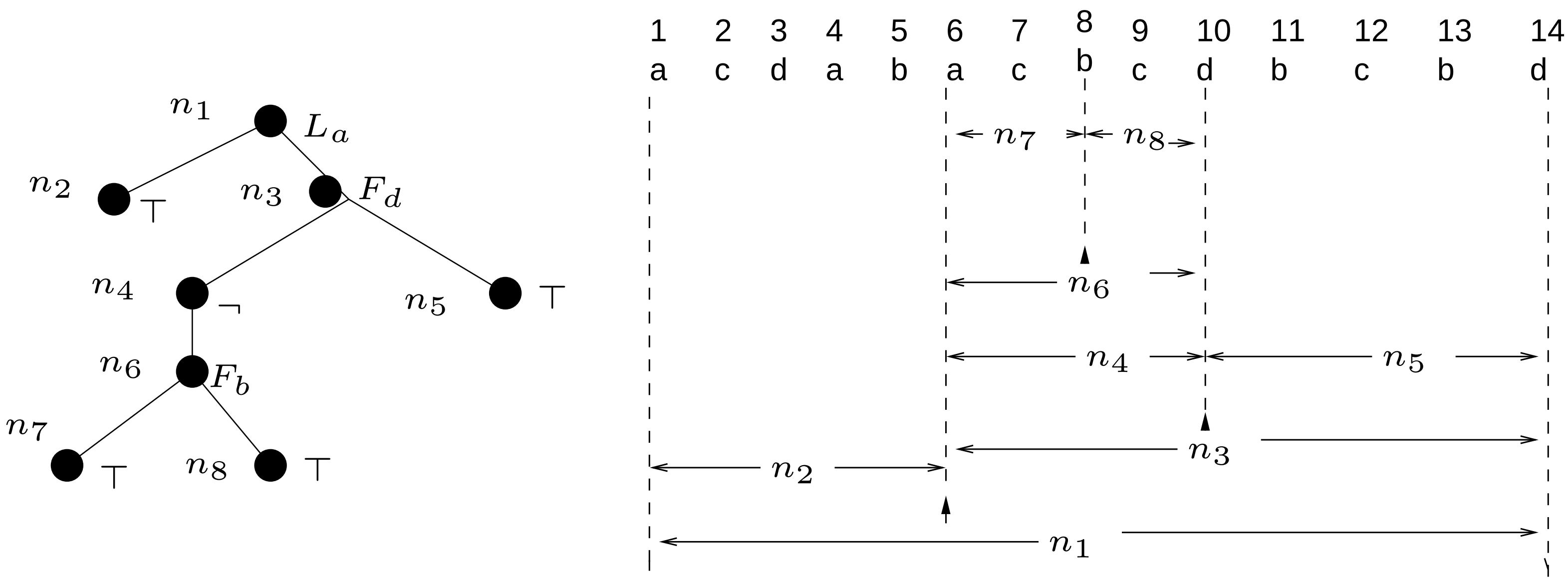
Formula $D \stackrel{\text{def}}{=} (\top L_a((\neg(\top F_b \top))F_d \top))$





Example

Formula $D \stackrel{\text{def}}{=} (\top L_a((\neg(\top F_b \top)) F_d \top))$



$w, [6, 14] \models n_4\ F_d\ n_5$ iff $w, [6, 10] \models n_4$ and $w, [10, 14] \models n_5$.

Unique parsability

Given a word w,

- Each node has unique associated interval (or none).

  $$Intv_w : Nodes \to INTV(w) \cup \{\mathbf{u}\}$$

  $Intv_w(n)$ depends only on the context of n.

- Each node of the form F_a or L_a has a unique “chopping” position (or none).

  $$cPos_w : MNodes \to INTV(w) \cup \{\mathbf{u}\}$$

- Truth of a node in its unique interval

  $$Eval_w : Nodes \to \{\mathbf{t}, \mathbf{f}, \mathbf{u}\}.$$

Main results

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- Unique parsability
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Left and right endpoint scanners

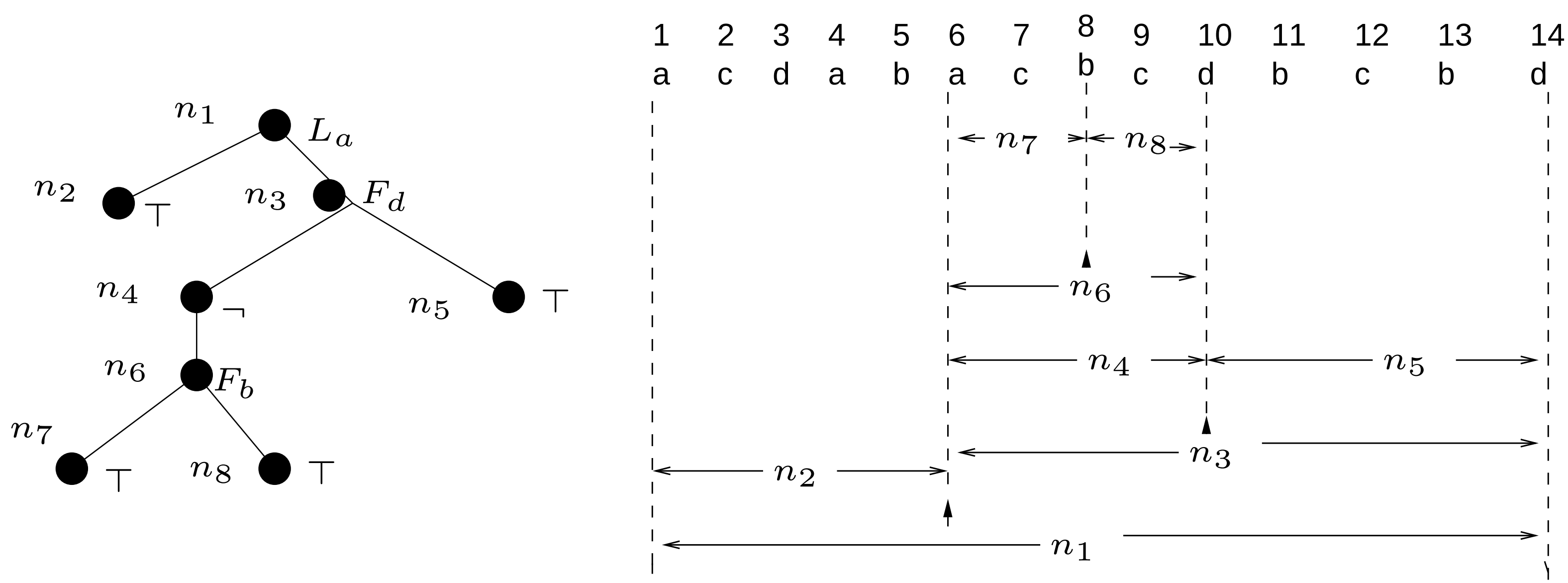
For each node n construct $po2dfa$

- $\mathcal{L}(n)$ which seeks and accepts at left endpoint of $Intv_w(n)$.
- $\mathcal{R}(n)$ which seeks and accepts at right endpoint of $Intv_w(n)$.
- Inductive construction based on depth of the node from $root$.

Lemma If $Intv_w(n) = [i, j]$ then $\mathcal{L}(n)(w, *) = (t, i)$ and $\mathcal{R}(n)(w, *) = (t, j)$.

Example endpoint scanners

Formula $D \stackrel{\text{def}}{=} (\top L_a((\neg(\top F_b \top)) F_d \top))$



- $\mathcal{L}(n_4) \stackrel{\text{def}}{=} \stackrel{\triangleleft}{\longrightarrow};\ \stackrel{a}{\longleftarrow}$
- $\mathcal{R}(n_4) \stackrel{\text{def}}{=} \mathcal{L}(n_4);\ \stackrel{d}{\longrightarrow}$.

Subformula automaton

Theorem For each node n we construct $po2dfa$ $\mathcal{M}(n)$ such that

- If $Eval_w(n) = t$ then $\mathcal{M}(n)(w, *) = (t, j)$ for some j.
- If $Eval_w(n) = f$ then $\mathcal{M}(n)(w, *) = (r, j)$ for some j.

Construction of $\mathcal{M}(n)$ By structural Induction.

- $\mathcal{M}(\top) \quad = \quad Acc$.
- $\mathcal{M}(\neg n) \quad = \quad \mathcal{M}(n)?Rej, Acc$.
- $\mathcal{M}(n_1 \vee n_2) \quad = \quad \mathcal{M}(n_1)?Acc, \mathcal{M}(n_2)$.

Automaton for F_a construct

Let $n = n_1 \; F_a \; n_2$. Then $\mathcal{M}(n) \;\; =$

$$\mathcal{L}(n); \xrightarrow{a};$$

Check if head is in $Intv_w(n)$;

$$\mathcal{M}(n_1); \;\; \mathcal{M}(n_2)$$

Main difficulty: How to check if current head position is in $Intv_w(n)$?

Proof: Carry the context around and use it.

Converse

- Effective construction $D: \ po2dfa \rightarrow UITL$ such that $L(A) = L(A(D))$. The size $|D(A)| \ = \ \mathcal{O}(2^{|A|})$.
- Proof idea: Describe the run of the $po2dfa$ in $UITL$.
- Proof: Carry the context around and use it. This is considerably more tedious than in the forward direction and yields a disjunction over many cases which in general gives an exponential upper bound for the size of the formula.
- Open: A logic $UITL^+$ which can polynomially (even linearly) encode $po2dfa$?

Summary

- A subclass of starfree expressions with the same expressive power as the unambiguous languages of Schützenberger (1976).
- Closing the piecewise testable languages with left- and right-deterministic operators has the power of unambiguous closure. Independently proved by Kufleitner and Weil (2008) using algebraic techniques.
- A "deterministic" logic admitting unique parsing of its models.
- A tractable fragment of ITL giving size $O(n^2)$ $po2dfa$ construction and polynomial time modelchecking.
- Expressively weak. To describe an automaton of n states requires an $O(2^n)$ length formula.

More open questions

Reg. Exp.	Membership	Emptiness	Interval TL	Modelchecking	Satisfiability
SF	PSpace	Nonelem.	ITL	PSpace	Nonelementary
USF	LogDCFL	PSpace	UITL	Ptime	PSpace

- An interval logic expressively equivalent to $FO^2[<, S]$? Its automata and expressiveness?
- There is an infinite quantifier alternation hierarchy in $FO[<]$, for which the only (not decidable) characterization known is in terms of semidirect products of monoids. It is known that $FO^2[<]$ corresponds to the second level (more precisely $\Delta_2[<]$). Where do we lose polynomial time?
- Can we tighten the NC^1 lower bound for membership/modelchecking (the same as for propositional logic)?

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